Chapter Resources

Lesson 2-1

Get Ready for the Lesson

Read the introduction to Lesson 2-1 in your textbook.
- Refer to the table. What does the ordered pair (8, 20) tell you? For a deer, the average longevity is 8 years and the maximum longevity is 20 years.
- Suppose that this table is extended to include more animals. Is it possible to have an ordered pair for the data in which the first number is larger than the second? Sample answer: No, the maximum longevity must always be greater than the average longevity.

Read the Lesson

1. a. Explain the difference between a relation and a function. Sample answer: A relation is any set of ordered pairs. A function is a special kind of relation in which each element of the domain is paired with exactly one element in the range.
   b. Explain the difference between domain and range. Sample answer: The domain of a relation is the set of all first coordinates of the ordered pairs. The range is the set of all second coordinates.

2. a. Write the domain and range of the relation shown in the graph.
   D: {(-3, -2, -1, 0, 3)}; R: {(-5, -4, 0, 1, 2, 4)

   b. Is this relation a function? Explain. Sample answer: No, it is not a function because one of the elements of the domain, 3, is paired with two elements of the range.

Remember What You Learned

3. Look up the words dependent and independent in a dictionary. How can the meaning of these words help you distinguish between independent and dependent variables in a function? Sample answer: The variable whose values depend on, or are determined by, the values of the other variable is the dependent variable.
2-1 Study Guide and Intervention

Relations and Functions

Graph Relations
A relation can be represented as a set of ordered pairs or as an equation; the relation is then the set of all ordered pairs (x, y) that make the equation true. The domain of a relation is the set of all first coordinates of the ordered pairs, and the range is the set of all second coordinates.

A function is a relation in which each element of the domain is paired with exactly one element of the range. You can tell if a relation is a function by graphing, then using the vertical line test. If a vertical line intersects the graph at more than one point, the relation is not a function.

Example
Graph the equation y = 2x – 3 and find the domain and range. Is the equation discrete or continuous? Does the equation represent a function?

Make a table of values to find ordered pairs that satisfy the equation. Then graph the ordered pairs.

The domain and range are both all real numbers. The equation can be graphed by line, so it is continuous. The graph passes the vertical line test, so it is a function.

Exercises
Graph each relation or equation and find the domain and range. Next determine if the relation is discrete or continuous. Then determine whether the relation or equation is a function.

1. (1, 3), (-3, 5), (-2, 5), (2, 3)
2. (3, -4), (1, 0), (-2, -3), (3, 2), (5, 1)
3. (0, 4), (-3, -2), (3, 2), (5, 1)

Exercise answers:
1. D = (-3, -2, 1, 2), R = (3, 5); discrete; yes
2. D = (1, 2, 3), R = (-4, -2, 2), discrete; no
3. D = (-3, 0, 3, 5), R = (-2, 1, 2, 4), discrete; yes
4. y = x^2 - 1
5. y = -x - 4
6. y = 3x + 2

Find each value if f(x) = -2x + 4.
1. f(12) -20 2. f(6) -8 3. f(2h) -4b + 4

Find each value if g(x) = x^3 - x.
4. g(5) 120 5. g(-2) -6 6. g(3) 243c^3 - 7c

Find each value if (x) = 2x + 2 and g(x) = 0.4x^2 - 1.2.
7. f(0.5) 5 8. f(-8) -16 1/4 9. g(3) 2.4
10. g(-2.5) 1.3 11. f(4a) 8a + 1 1/2 12. g(b) 2a 10 - 1.2
13. f(1/3) 6 2/3 14. g(10) 38.8 15. f(200) 400.01

16. Find the values of f(2) and f(5). f(2) = 7, f(5) = 49
17. Compare the values of f(2) · f(5) and f(2 · 5). f(2) · f(5) = 343, f(2 · 5) = 199
### Skills Practice

**Relations and Functions**

Determine whether each relation is a function. Write yes or no.

1. Domain: no; Range: yes

2. Domain: yes; Range: yes

3. Domain: no; Range: no

4. Domain: yes; Range: yes

Determine if the equation is a function. Write yes or no.

5. \( x = 3y \) (\( x \) is a function of \( y \))

6. \( y = 2x - 1 \)

Graph each relation or equation and find the domain and range. Then determine if the relation or equation is a function.

7. \( D = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\} \) discrete; yes

8. \( D = \{(1, 2), (2, 4), (3, 6), (4, 8)\} \) continuous; yes

9. \( R = \{(1, 2), (2, 3), (3, 4), (4, 5)\} \) discrete; yes

Find the value of each function.

10. \( f(x) = 3x - 2 \)

11. \( g(x) = x^2 \)

### Relations and Functions

**Graph each relation or equation and find the domain and range.**

**Next determine if the relation or equation is a function.**

**Domain:**

- \( D = \{(1, 16), (2, 16), (3, 32), (4, 32), (5, 48)\} \)
- \( D = \{(x, y) | x \in \mathbb{R}, y \in \mathbb{R}\} \)

**Range:**

- \( R = \{y | y = 3x - 2\} \)
- \( R = \{y | y = x^2\} \)

**Function:**

- Yes
- No
- Undefined

**Continuous:**

- Yes
- No

**Discrete:**

- Yes
- No

### Computing

- If a computer can do one calculation in 0.0000000015 second, then the computer can do 6.5 billion calculations in 7.5 seconds.
1. PLANETS The table below gives the mean distance from the Sun and orbital period of the nine major planets in our Solar System. Think of the mean distance as the domain and the orbital period as the range of a relation. Is this relation a function? Explain.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Mean Distance from Sun (AU)</th>
<th>Orbital Period (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0.387</td>
<td>0.241</td>
</tr>
<tr>
<td>Venus</td>
<td>0.723</td>
<td>0.615</td>
</tr>
<tr>
<td>Earth</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Mars</td>
<td>1.524</td>
<td>1.881</td>
</tr>
<tr>
<td>Jupiter</td>
<td>5.204</td>
<td>11.75</td>
</tr>
<tr>
<td>Saturn</td>
<td>9.582</td>
<td>29.5</td>
</tr>
<tr>
<td>Uranus</td>
<td>19.201</td>
<td>84</td>
</tr>
<tr>
<td>Neptune</td>
<td>30.047</td>
<td>165</td>
</tr>
<tr>
<td>Pluto</td>
<td>39.236</td>
<td>248</td>
</tr>
</tbody>
</table>

Yes, it is a function because every value in the domain corresponds to a single value in the range.

2. PROBABILITY Martha rolls a number cube several times and makes the frequency graph shown. Write a relation to represent this data.

{1, 2, 3, 4, 5, 6} \times \{1, 2, 3, 4, 5, 6\}

3. SCHOOL The number of students N in Vasai's school is given by N = 120 + 30G, where G is the grade level. Is 285 in the range of this function? No, if \( N = 285 \), then \( G = 5.5 \) but grade levels are integers so 285 is not in the range of the function.

4. FLOWERS Anthony decides to decorate a ballroom with \( r = 3p + 20 \) roses, where \( p \) is the number of pairs. That is, \( a = 2p \), where \( a \) is the number of pairs. What is \( r \) as a function of \( p \)?

\[ r = 6p + 20 \]

5. Graph the data.

6. Identify the domain and range.
   - Domain: \( \{1, 2, 3, 4, 5, 6\} \)
   - Range: \( \{8, 10, 15, 22, 31, 44\} \)

   Yes, it is a function because every value in the domain corresponds to a single value in the range.

8. Can a set be mapped onto a set with fewer elements than it has? yes

9. Can a set be mapped onto a set with more elements than it has? yes

10. Can a set have a one-to-one mapping into a set that has more elements than it has? yes

11. If a mapping from set A into set B is bijective, what can you conclude about the number of elements in A and B? The sets have the same number of elements.
Get Ready for the Lesson

Read the introduction to Lesson 2-2 in your textbook.

• If Lolita spends \( \frac{2}{3} \) hours studying math, how many hours will she have to study chemistry? \( \frac{1}{2} \) hours
• Suppose that Lolita decides to stay up one hour later so that she now has 5 hours to study and do homework. Write a linear equation that describes this situation.

\[ x + y = 5 \]

Read the Lesson

1. Write yes or no to tell whether each linear equation is in standard form. If it is not, explain why it is not.
   a. \( -x + 2y = -5 \) No; \( A \) is negative.
   b. \( 9x - 12y = -5 \) Yes
   c. \( 5x - 7y = 3 \) Yes
   d. \( 2x - \frac{4}{3}y = 1 \) No; \( B \) is not an integer.
   e. \( 0x + 9y = 0 \) No; \( A \) and \( B \) are both 0.
   f. \( 2x + 4y = 8 \) No; The greatest common factor of 2, 4, and 8 is 2, not 1.

2. How can you use the standard form of a linear equation to tell whether the graph is a horizontal line or a vertical line? If \( A = 0 \), then the graph is a horizontal line. If \( B = 0 \), then the graph is a vertical line.

Remember What You Learned

3. One way to remember something is to explain it to another person. Suppose that you are studying this lesson with a friend who thinks that she should let \( x = 0 \) to find the \( x \)-intercept and let \( y = 0 \) to find the \( y \)-intercept. How would you explain to her how to remember the correct way to find intercepts of a line? Sample answer: The \( x \)-intercept is the \( x \)-coordinate of a point on the \( x \)-axis. Every point on the \( x \)-axis has \( y \)-coordinate 0, so let \( y = 0 \) to find an \( x \)-intercept. The \( y \)-intercept is the \( y \)-coordinate of a point on the \( y \)-axis. Every point on the \( y \)-axis has \( x \)-coordinate 0, so let \( x = 0 \) to find a \( y \)-intercept.
2-2 Study Guide and Intervention (continued)

Linear Equations

Example: Write each equation in standard form. Identify A, B, and C.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Standard Form</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>y = 8x - 5</td>
<td>y = 8x - 5</td>
<td>8</td>
<td>1</td>
<td>-5</td>
</tr>
<tr>
<td>4x - 7y + 21</td>
<td>4x - 7y + 21</td>
<td>4</td>
<td>-7</td>
<td>21</td>
</tr>
</tbody>
</table>

Exercises: Write each equation in standard form. Identify A, B, and C.

1. 2x - 4y = -1; A = 2, B = -4, C = -1
2. 2x - 5y + 3 = 0; A = 2, B = 5, C = 3
3. 3x - 5y + 2 = 0; A = 3, B = -5, C = 2

State whether each equation or function is linear. Write yes or no. If no, explain your reasoning.

1. y = 3x; yes
2. y = -2 + 5x; yes
3. 2x + y = 10; yes
4. f(x) = 4x^2; no; the exponent of x is not 1
5. \(\frac{3}{2}x + y = 15\); yes
6. \(\frac{1}{3}x = y + 8\); no; x is in a denominator.
7. g(x) = 8; no; x is inside a square root.

Find the x-intercept and the y-intercept of the graph of each equation. Then graph the equation.

15. y = 3x - 6; 2, -6
16. y = -2x; 0, 0
17. x + y = 5; 5, 5
18. 2x + 5y = 10; 5, 2
2-2 Practice

Linear Equations

State whether each equation or function is linear. Write yes or no. If no, explain your reasoning.

1. \( h(x) = 23 \)  yes
2. \( y = \frac{2}{3}x \)  yes
3. \( y = \frac{3}{x} \)  No; \( x \) is a denominator.
4. \( 9 - 5x = 2 \)  No; \( x \) and \( y \) are multiplied.

Write each equation in standard form. Identify \( A, B, \) and \( C. \)

5. \( y = 7x - 5 \)  \( 3x - y = 1, 5 \)
6. \( y = \frac{3}{2}x + 5 \)  \( 3x - 8y = -40; 3, -8, -40 \)
7. \( 3y - 5 = 0 \)  \( 3y = 5; 0, 3, 5 \)
8. \( x = -\frac{2}{3}y + \frac{3}{4} \)  \( 28x + 8y = 21; 28, 8, 21 \)

Find the \( x \)-intercept and the \( y \)-intercept of the graph of each equation. Then graph the equation.

9. \( y = 2x + 4 \)  \(-2, 4 \)
10. \( 2x + 7y = 14 \)  \( 7, 2 \)
11. \( y = -2x - 4 \)  \(-2, -4 \)
12. \( 6x + 2y = 6 \)  \( 1, 3 \)

13. **MEASURE** The equation \( y = 2.54x \) gives the length in centimeters corresponding to a length \( x \) in inches. What is the length in centimeters of a 1-foot ruler? **30.48 cm**

**LONG DISTANCE** For Exercises 14 and 15, use the following information.

For Meg’s long-distance calling plan, the monthly cost \( C \) in dollars is given by the linear function \( C(t) = 6 + 0.05t \), where \( t \) is the number of minutes talked.

14. What is the total cost of talking 8 hours? Of talking 20 hours? **$30; $66**
15. What is the effective cost per minute (the total cost divided by the number of minutes talked) of talking 8 hours? Of talking 20 hours? **$0.0625; $0.055**

2-2 Word Problem Practice

Linear Equations

1. **WORK RATE** The linear equation \( n = 100 \) describes \( n \), the number of origami boxes that Holly can fold in \( t \) hours. How many boxes can Holly fold in 3 hours? **30 boxes**

2. **BASKETBALL** Tony tossed a basketball. Below is a graph showing the height of the basketball as a function of time. Is this the graph of a linear function? Explain.

   No, it is not linear because graphs of linear functions are always straight lines. This graph curves.

3. **PROFIT** Paul charges people $25 to test the air quality in their homes. The device he uses to test air quality cost him $500. Write an equation that describes Paul’s net profit as a function of the number of clients he gets. How many clients does he need to break even? Paul’s profit is \( p = 25c - 500 \), if \( c \) is the number of clients and \( p \) is his profit. He needs 20 clients to break even.

4. **RAMP** A ramp is described by the equation \( 5x + 7y = 35 \). What is the area of the shaded region? **17.5 square units**

5. Write an equation that relates \( x \) and \( y \). Sample answer: \( 2x + 2y + 10 = 110 \)

6. Write the linear equation from Exercise 5 in standard form. \( x + y = 50 \)

7. Graph the equation.
Diophantine Equations

The first great algebraist, Diophantus of Alexandria (about A.D. 300), devoted much of his work to the solving of indeterminate equations. An indeterminate equation has more than one variable and an unlimited number of solutions. An example is \( x + 2y = 4 \).

When the coefficients of an indeterminate equation are integers and you are asked to find solutions that must be integers, the equation is called a diophantine. Such equations can be quite difficult to solve, often involving trial and error—and some luck!

Solve each diophantine equation by finding at least one pair of positive integers that makes the equation true. Some hints are given to help you.

1. \( 2x + 5y = 32 \)
   a. First solve the equation for \( x \). \( x = 16 - \frac{5y}{2} \)
   b. Why must \( y \) be an even number? If \( y \) is odd, then \( x \) is not an integer.
   c. Find at least one solution. Sample answers: (11, 2), (6, 4), (1, 6)

2. \( 5x + 2y = 42 \)
   a. First solve the equation for \( x \). \( x = 8 - \frac{2y}{5} \)
   b. Rewrite your answer in the form \( x = \) some expression. \( x = 8 - \frac{2y}{5} \)
   c. Why must \( \left(\frac{2}{5}, 2y\right) \) be a multiple of 5? So that \( x \) is an integer
   d. Find at least one solution. Sample answers: (8, 1), (6, 4), (11, 2, 16)

3. \( 2x + 7y = 29 \)
   (11, 1) or (4, 3)

4. \( 7x + 5y = 118 \)
   (14, 4), (9, 11) or (4, 18)

5. \( 8x - 13y = 100 \)
   (19, 4), (32, 12), or any \((x, y)\) where \( y = 4n \) and \( n \) is a positive odd integer

6. \( 3x + 4y = 22 \)
   (6, 1) or (2, 4)

7. \( 5x - 14y = 11 \)
   (5, 1), (19, 6), or any \((x, y)\) where \( y = 5m \) and \( m \) is a positive integer

8. \( 7x + 3y = 40 \)
   (4, 4) or (1, 11)
Study Guide and Intervention

Slope

For points \((x_1, y_1)\) and \((x_2, y_2)\), where \(x_1 \neq x_2\),

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

**Example 1** Determine the slope of the line that passes through \((2, 1)\) and \((-4, 5)\).

\[
m = \frac{5 - 1}{-4 - 2} = \frac{4}{-6} = -\frac{2}{3}
\]

The slope of the line is \(-\frac{2}{3}\).

**Example 2** Graph the line passing through \((-1, -3)\) with a slope of \(\frac{1}{2}\).

Graph the ordered pair \((-1, -3)\). Then, according to the slope, go up 1 unit and right 2 units. Plot the new point \((1, -2)\). Connect the points and draw the line.

**Exercises**

Find the slope of the line that passes through each pair of points.

1. \((4, 7)\) and \((6, 13)\)
2. \((2, 6)\) and \((3, 4)\)
3. \((5, 1)\) and \((7, -3)\)
4. \((-5, -3)\) and \((-4, 3)\)
5. \((5, 0)\) and \((-1, -2)\)
6. \((-1, -4)\) and \((-13, 2)\)
7. \((7, -2)\) and \((3, 3)\)
8. \((-5, 9)\) and \((5, 5)\)
9. \((-4, -2)\) and \((-4, -8)\)

Graph the line passing through the given point with the given slope.

10. slope = \(\frac{1}{3}\) passes through \((0, 2)\)
11. slope = \(2\) passes through \((1, 4)\)
12. slope = \(0\) passes through \((-2, -5)\)
13. slope = \(-1\) passes through \((-4, 6)\)
14. slope = \(-\frac{3}{4}\) passes through \((-3, 0)\)
15. slope = \(\frac{1}{5}\) passes through \((0, 0)\)

**Parallel and Perpendicular Lines**

In a plane, nonvertical lines with the same slope are parallel. All vertical lines are parallel.

In a plane, two oblique lines are perpendicular if and only if the product of their slopes is \(-1\). Any vertical line is perpendicular to any horizontal line.

**Example** Are the line passing through \((2, 6)\) and \((-2, 2)\) and the line passing through \((3, 0)\) and \((0, 4)\) parallel, perpendicular, or neither?

Find the slopes of the two lines.

The slope of the first line is \(\frac{6 - 2}{2 - (-2)} = 1\).

The slope of the second line is \(\frac{0 - 4}{0 - 3} = \frac{4}{3}\).

The slopes are not equal and the product of the slopes is not \(-1\), so the lines are neither parallel nor perpendicular.

**Exercises**

Are the lines parallel, perpendicular, or neither?

1. the line passing through \((4, 3)\) and \((1, -3)\) and the line passing through \((1, 2)\) and \((-1, 3)\) neither
2. the line passing through \((2, 8)\) and \((-2, 2)\) and the line passing through \((0, 9)\) and \((6, 0)\) parallel
3. the line passing through \((3, 9)\) and \((-2, -1)\) and the graph of \(y = x\) perpendicular
4. the line with \(x\)-intercept \(-2\) and \(y\)-intercept 5 and the line with \(x\)-intercept 2 and \(y\)-intercept \(-5\) parallel
5. the line with \(x\)-intercept 1 and \(y\)-intercept 3 and the line with \(x\)-intercept 3 and \(y\)-intercept 1 neither
6. the line passing through \((-2, -3)\) and \((2, 5)\) and the graph of \(x + 2y = 10\) perpendicular
7. the line passing through \((-4, -8)\) and \((6, -4)\) and the graph of \(2x - 5y = 5\) parallel
2-3 Skills Practice

Slope

Find the slope of the line that passes through each pair of points.
1. (1, 5), (3, 3) 4. (-5, 4, 2) -3 2
2. (0, 2), (3, 0) -2 3. (1, 9), (0, 6) 3
5. (-3, 5), (-3, -1) undefined
6. (-2, -2), (10, -2) 0

Graph the line passing through the given point with the given slope.
7. (0, 3), m = 3
8. (-2, 1), m = -3
9. (0, 2), m = 0
10. (2, -3), m = 4

Graph the line that satisfies each set of conditions.
11. passes through (3, 0), perpendicular to a line whose slope is \( \frac{3}{2} \)
12. passes through (-3, -1), parallel to a line whose slope is -1

DEPRECIATION For Exercises 13–15, use the following information.
A machine that originally cost $15,600 has a value of $7,500 at the end of 3 years. The same machine has a value of $8,600 at the end of 8 years.

13. Find the average rate of change in value (depreciation) of the machine between its purchase and the end of 3 years. \(-\$2,700 \text{ per year}\)
14. Find the average rate of change in value of the machine between the end of 3 years and the end of 8 years. \(-\$940 \text{ per year}\)
15. Interpret the sign of your answers. It is negative because the value is decreasing.

Hiking Naomi left from an elevation of 7,400 feet at 7:00 A.M. and hiked to an elevation of 9,800 feet by 11:00 A.M. What was her rate of change in altitude? \(600 \text{ ft/h}\)
Word Problem Practice

Slope

1. TETHER A tether is tied tautly to the top of a pole as shown. What is the slope of the tether?

2. AVIATION An airplane descends along a straight-line path with a slope of -0.1 to land at an airport. Use the information in the diagram to determine the initial height of the airplane.

3. ROCK CLIMBING The table below shows Gail's altitude above ground during a rock climb up a cliff. Complete the following table of Gail's average rate of ascent.

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Average rate of ascent (m/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10:00-10:20</td>
<td>66</td>
</tr>
<tr>
<td>10:20-10:40</td>
<td>24</td>
</tr>
<tr>
<td>10:40-11:00</td>
<td>9</td>
</tr>
</tbody>
</table>

4. DESIGN An architect is designing a window with slanted interior bars. The crossbeam is perpendicular to the other four bars. What is the slope of the crossbeam?

5. Find the average number of pages Bridget read per day.

6. On which days did Bridget read more pages than her daily average?

Day 2 and Day 3

7. If Bridget had been able to keep up the pace she had on day 3, how many days would it have taken her to finish the book?

5 days

Enrichment

The Increase in Greenhouse Gases

The atmosphere is composed of about 50% carbon dioxide, CO₂. The levels of carbon dioxide are increasing due to increased fuel consumption and housing and commercial development. The concentration of a compound is measured in parts per million (ppm). For example, if there were 500 CO₂ molecules out of one million air particles, then the CO₂ level would be 500 ppm.

1. In 1965, the concentration of CO₂ was 320 ppm. In 2004, the concentration was 378 ppm. Determine the rate at which CO₂ increased in ppm per year.

   CO₂ is increased approximately 1.487 ppm per year.

2. Carbon dioxide concentration is related to human consumption of fossil fuels and the decrease of trees due to development, therefore an increase in human population will result in an increase in carbon dioxide. In 1980 the U.S. population was 225 million. The 2000 census reported 281 million. At what rate is the population increasing per year?

   U.S. Population is increasing about 2.8 million people per year. Therefore, the population in 2006 would be about 297.8 million.

3. Use the figures from Exercises 1 and 2 to determine about how much CO₂ is "produced" per million people. Is it possible to reduce the concentration of carbon dioxide in the atmosphere when the human population is increasing? Explain.

   0.531 ppm per million people; Yes, it is possible; humans would have to reduce their consumption of fossil fuels.

4. The greenhouse effect is heat "trapped" by gases such as carbon dioxide, which acts as a "blanket" for the earth. Higher concentration levels of carbon dioxide amplify the greenhouse effect. Thus, global temperature is related to the concentration of CO₂. Records indicate that the increase in global temperature since 1940 is 0.02 degrees Fahrenheit per year. Each degree rise in temperature causes ocean levels to rise one-half a foot. Use your data to determine in what year the ocean level will rise 2 feet. What impact will this have on coastal regions of the United States?

   Current ocean levels will rise 2 feet in 200 years. This would decrease beaches and possibly flood some coastal cities.
and is the slope and 2 is the slope. Simplify. Subtract from both sides.

\[ y = mx + b \]

Write the point-slope form of the equation of a line. Then explain the meaning of each \( m \) and \( b \) in the equation. Glencoe Algebra 2

**Example 2**

Write an equation in slope-intercept form for each graph.

1. \( y = -2x - 2 \)
2. \( y = 3x + 4 \)
3. \( y = x + 3 \)
4. \( y = -3x + 9 \)

**Exercises**

Write the point-slope form of the equation of a line.

1. \( y = \frac{1}{2}x + 4 \)
2. \( y = \frac{2}{3}x + 4 \)
3. \( y = \frac{1}{2}x + 3 \)
4. \( y = \frac{2}{3}x + 4 \)
5. \( y = \frac{1}{2}x + 3 \)

**Answers** (Lesson 2-4)

*Sample answer:*

Suppose that your algebra teacher asks you to write the point-slope form of the equation of a line. Then explain the meaning of each \( m \) and \( b \) in the equation. Glencoe Algebra 2

If the total cost of producing a product is given by the equation \( C = 5400 + 1.37x \), where \( x \) is the variable cost for each item produced, \( \$5400 \) is the fixed cost, and \( \$1.37 \) is the variable cost of \( \$82,750 \) to produce each item. Add 6 to both sides.

The slope-intercept form is \( y = mx + b \). Multiply both sides of this equation by \( \frac{1}{2} \). Subtract from both sides.

If the total cost of producing a product is given by the equation \( C = 5400 + 1.37x \), where \( x \) is the variable cost for each item produced, \( \$5400 \) is the fixed cost, and \( \$1.37 \) is the variable cost of \( \$82,750 \) to produce each item. Add 6 to both sides.

The slope-intercept form is \( y = mx + b \). Multiply both sides of this equation by \( \frac{1}{2} \). Subtract from both sides.

If the total cost of producing a product is given by the equation \( C = 5400 + 1.37x \), where \( x \) is the variable cost for each item produced, \( \$5400 \) is the fixed cost, and \( \$1.37 \) is the variable cost of \( \$82,750 \) to produce each item. Add 6 to both sides.

The slope-intercept form is \( y = mx + b \). Multiply both sides of this equation by \( \frac{1}{2} \). Subtract from both sides.
### 2-4 Writing Linear Equations

#### Parallel and Perpendicular Lines
Use the slope-intercept or point-slope form to find equations of lines that are parallel or perpendicular to a given line. Remember that parallel lines have equal slope, and that perpendicular lines have slopes that are negative reciprocals of each other.

**Example 1**
Write an equation of the line that passes through (0, 2) and is perpendicular to the line whose equation is \( y = -\frac{1}{2}x + 3 \).

The slope of the given line is \(-\frac{1}{2}\). The slope of the line perpendicular to this line is the negative reciprocal of \(-\frac{1}{2}\), which is \(2\). Using the point-slope form and the given point (0, 2), we can write the equation:

\[
(0) - 2 = m(0) - x + 3
\]

\[
y - 2 = 2x - 8
\]

An equation of the line is \( y = 2x - 14 \).

**Example 2**
Write an equation of the line that passes through \((-1, 5)\) and is parallel to the graph of \( y = 3x + 1 \).

The slope of the given line is 3. Since the slopes of parallel lines are equal, the slope of the parallel line is also 3. Use the point and the given point to write the equation:

\[
y - 5 = 3(x + 1)
\]

\[
y = 3x + 8
\]

An equation of the line is \( y = 3x + 8 \).

### Exercises

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

1. passes through \((-4, 2)\), parallel to the line whose equation is \( y = \frac{1}{2}x + 5 \)
   \( y = \frac{1}{2}x + 4 \)

2. passes through \((3, 1)\), perpendicular to the graph of \( y = -3x + 2 \)
   \( y = \frac{1}{3}x \)

3. passes through \((1, -1)\), parallel to the line that passes through \((4, 1)\) and \((2, -3)\)
   \( y = 2x - 3 \)

4. passes through \((4, 7)\), perpendicular to the line that passes through \((3, 6)\) and \((3, 15)\)
   \( y = 7 \)

5. passes through \((8, -6)\), parallel to the graph of \( 2x - y = 4 \)
   \( y = \frac{1}{2}x - 2 \)

6. passes through \((2, -2)\), perpendicular to the graph of \( x + 5y = 6 \)
   \( y = 5x - 12 \)

7. passes through \((6, 1)\), parallel to the line with \( x\)-intercept \(-3\) and \( y\)-intercept 5
   \( y = \frac{3}{2}x - 9 \)

8. passes through \((-2, 1)\), perpendicular to the line \( y = 4x - 11 \)
   \( y = -\frac{1}{4}x + \frac{1}{2} \)

### Skills Practice

**Writing Linear Equations**

State the slope and \( y\)-intercept of the graph of each equation.

1. \( y = 7x - 5 \)
   \( m = 7, b = -5 \)

2. \( y = -\frac{3}{2}x + 3 \)
   \( m = -\frac{3}{2}, b = 3 \)

3. \( y = \frac{2}{3}x + 0 \)
   \( m = \frac{2}{3}, b = 0 \)

4. \( 3x + 4y = 4 - \frac{3}{4} \)
   \( m = -\frac{3}{4}, b = \frac{3}{2} \)

5. \( 7y = 4x - 7 \)
   \( m = \frac{4}{7}, b = -1 \)

6. \( 3x - 2y + 6 = 0 \)
   \( m = \frac{3}{2}, b = 3 \)

7. \( 2x - y = 5 \)
   \( m = 2, b = -5 \)

8. \( 2y = 6 - 5x \)
   \( m = -\frac{5}{2}, b = 3 \)

Write an equation in slope-intercept form for each graph.

9. \( y = 3x - 1 \)

10. \( y = -1 \)

11. \( y = -2x + 3 \)

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

12. slope 3, passes through \((1, -3)\)
    \( y = 3x - 6 \)

13. slope \(-1\), passes through \((0, 0)\)
    \( y = -x \)

14. slope \(-2\), passes through \((0, -5)\)
    \( y = -2x - 5 \)

15. slope 3, passes through \((2, 0)\)
    \( y = 3x - 6 \)

16. passes through \((1, -2)\) and \((-3, 1)\)
    \( y = -\frac{3}{2}x - \frac{7}{2} \)

17. passes through \((-2, -4)\) and \((1, 8)\)
    \( y = 4x + 4 \)

18. \( x\)-intercept 2, \( y\)-intercept -6
    \( y = 3x - 6 \)

19. \( x\)-intercept \(\frac{5}{2}\), \( y\)-intercept 5
    \( y = -2x + 5 \)

20. passes through \((3, -1)\), perpendicular to the graph of \( y = -\frac{1}{3}x - 4 \)
    \( y = 3x - 10 \)
2.4 Practice Writing Linear Equations

State the slope and y-intercept of each equation.

1. \( y = 8x + 12 \)
2. \( y = 0.25x - 1 \)
3. \( y = -\frac{3}{5}x - \frac{3}{5} \)
4. \( 3y = 70 \)
5. \( 3x = -15 + 5y \)
6. \( 2x - 3y = 10 \)

Write an equation in slope-intercept form for each graph.

7. \( y = 2 \)
8. \( y = \frac{3}{2}x - 2 \)
9. \( y = -\frac{2}{3}x + 1 \)

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

10. slope -5, passes through (-3, -8)
11. slope \( \frac{4}{3} \), passes through (10, -3)
12. slope 0, passes through (0, -10)
13. slope \( -\frac{2}{3} \), passes through (6, -8)
14. passes through (3, 11) and (-6, 5)
15. passes through (7, -2) and (3, -1)
16. x-intercept 3, y-intercept 2
17. x-intercept -5, y-intercept 7
18. passes through (-8, -7), perpendicular to the graph of \( y = 4x - 3 \)
19. RESERVOIRS The surface of Grand Lake is at an elevation of 648 feet. During the current drought, the water level is dropping at a rate of 3 inches per day. If this trend continues, write an equation that gives the elevation in feet of the surface of Grand Lake after \( x \) days. \( y = -0.25x + 648 \)
20. BUSINESS Tony Marconi’s company manufactures CD-ROM drives. The company will make \$150,000 profit if it manufactures 100,000 drives, and \$1,750,000 profit if it manufactures 500,000 drives. The relationship between the number of drives manufactured and the profit is linear. Write an equation that gives the profit \( P \) when \( n \) drives are manufactured. \( P = 4n - 250,000 \)

2.4 Word Problem Practice Writing Linear Equations

1. HIKING Tim began a hike at the base of the mountain that is 129 feet above sea level. He is hiking at a steady rate of 5 feet per minute. Let \( t \) be the number of minutes he has been hiking. Write an equation in slope-intercept form that represents how many feet above sea level Tim has hiked.
   \( A = 5t + 129 \)

2. CHARITY By midnight, a charity had collected 83 shirts. Every hour after that, it collected 20 more shirts. Let \( h \) be the number of hours since midnight and \( s \) be the number of shirts. Write a linear equation in slope-intercept form that relates the number of shirts collected and the number of hours since midnight.
   \( s = 20h + 83 \)

3. MAPS The post office and city hall are marked on a coordinate plane. Write the equation of the line in slope-intercept form that passes through these two points.

4. RIGHT TRIANGLES The line containing the base of a right triangle has the equation \( y = 3x + 4 \). The leg perpendicular to the base has an endpoint at (6, 1). What is the slope-intercept form of the equation of the line containing the leg?
   \( y = -\frac{1}{3}x + 3 \)

5. Using the lower left corner of the bulletin board as the origin, what is the equation of the line in slope-intercept form?
   \( y = \frac{1}{6}x + 2 \)

6. The students change their mind and decide that the line should be lowered by 1 foot. What is the equation of the lowered line in slope-intercept form?
   \( y = \frac{1}{6}x + 1 \)

7. What are the coordinates of the center of the bulletin board? Does the lowered line pass through the center? Explain.
   \( (3, 1.5); \) Yes, it does because the coordinates of the center satisfy the equation of the lowered line.

\( y = \frac{3}{2}x + 4.5 \)
Two-Intercept Form of a Linear Equation

You are already familiar with the slope-intercept form of a linear equation, 
\[ y = mx + b \]. Linear equations can also be written in the form \( \frac{x}{a} + \frac{y}{b} = 1 \) with 
the \( x \)-intercept \( a \) and \( y \)-intercept \( b \). This is called two-intercept form.

**Example 1**
Draw the graph of \( \frac{x}{3} + \frac{y}{6} = 1 \).
The graph crosses the \( x \)-axis at \(-3\) and the \( y \)-axis at \(6\). Graph \((-3, 0)\) and \((0, 6)\), then draw a straight line through them.

**Example 2**
Write \( 3x + 4y = 12 \) in two-intercept form.
Divide by 12 to obtain 1 on the right side.
\[ \frac{x}{4} + \frac{y}{3} = 1 \]
The \( x \)-intercept is \(4\); the \( y \)-intercept is \(3\).

**Exercises**
Use the given intercepts \( a \) and \( b \), to write an equation in two-intercept form. Then draw the graph. See students’ graphs.
1. \( a = -2, b = -4 \)
\[ \frac{x}{-2} + \frac{y}{-4} = 1 \]
2. \( a = -1, b = -8 \)
\[ \frac{x}{-1} + \frac{y}{-8} = 1 \]
3. \( a = 3, b = 5 \)
\[ \frac{x}{3} + \frac{y}{5} = 1 \]
4. \( a = 6, b = 9 \)
\[ \frac{x}{6} + \frac{y}{9} = 1 \]

Write each equation in two-intercept form. Then draw the graph.
5. \( 3x - 2y = -6 \)
\[ \frac{x}{-2} + \frac{y}{3} = 1 \]
6. \( \frac{2x}{3} + \frac{y}{4} = 1 \)
\[ \frac{x}{2} + \frac{y}{4} = 1 \]
7. \( 5x + 2y = -10 \)
\[ \frac{x}{-2} + \frac{y}{-5} = 1 \]

Spreadsheet Activity
Using Linear Equations

The slope intercept form of a linear equation is \( y = mx + b \), where \( m \) is the slope and \( b \) is the \( y \)-intercept. Recall that the formula for the slope of a line through \((x_1, y_1)\) and \((x_2, y_2)\) is \( m = \frac{y_2 - y_1}{x_2 - x_1} \). You can use the formula for slope and the slope-intercept form to find the value of \( b \).

**Example**
State the slope and \( y \)-intercept of the graph of the line through \((5, 2)\) and \((4, 1)\). Then write an equation of the line in slope-intercept form.

**Step 1** Use Columns A and B to represent the first point, and Columns C and D to represent the second point on the line. Enter the formula for slope in Column E.

**Step 2** Substitute one of the given points into the slope-intercept form and solve for \( b \). Since we know the slope of the line, we can solve for \( b \).

\[ y = mx + b \]
\[ y_1 = mx_1 + b \]
\[ y_1 - mx_1 = b \]

Solve for \( b \).

Enter this formula into Column F using the names of the spreadsheet cells.

The slope of the line through \((5, 2)\) and \((4, 1)\) is \( -1 \). Thus, the equation of the line slope-intercept form is \( y = -x + (-3) \) or \( y = -x - 3 \).

**Exercises**
Use a spreadsheet to find the slope and \( y \)-intercept of the line through each pair of points. Then write an equation of the line in slope-intercept form.
1. \((0, -5), (2, 5)\)
\[ y = 5x - 5 \]
2. \((4, 2), (-3, -5)\)
\[ y = -x - 2 \]
3. \((-1, -4), (1, 3)\)
\[ \frac{y}{2} - \frac{x}{2} = 1 \]
\[ y = 2x - 1 \]
4. \((-4, -9), (8, 3)\)
\[ y = -x + 5 \]
5. \((12, 9), (10, 10)\)
\[ y = -0.5x + 15 \]
\[ y = -0.5x + 2.35 \]
6. \((-1.5, 3.1), (0.9, 19)\)
\[ y = -0.5x + 2.35 \]

7. Does the spreadsheet work when two points have the same \( x \)-coordinates? Explain. No; The slope is undefined.
**Answers (Lesson 2-5)**

### Modeling Real-World Data: Using Scatter Plots

1. Suppose that a set of data can be modeled by a linear equation. Explain the difference between a scatter plot of the data and a graph of the linear equation that models that data.

   **Sample answer:** The scatter plot is a discrete graph. It is made up just of the individual points that represent the data points. The linear equation has a continuous graph that is the line that best fits the data points.

2. Suppose that tuition at a state college was $3800 per year in 2000 and has been increasing at a rate of $200 per year.

   a. Write a prediction equation that expresses this information.

   **Example:**
   
   \[ y = 3800 + 200x \]

   b. Explain the meaning of each variable in your prediction equation.

   **Sample answer:** The number of years since 2000 and $y$ represents the tuition in that year.

3. Use this model to predict the tuition at this college in 2007.

   **Answer:** $5200

4. Look up the word **scatter** in a dictionary. How can its definition help you to remember the meaning of the difference between a scatter plot and the graph of a linear equation?

   **Sample answer:** To **scatter** means to break up and go in many directions. The points on a scatter plot are broken up. In the graph of a linear equation, the points are connected to form a continuous line.
Chapter 2

Glencoe Algebra 2

Answers (Lesson 2-5)

Example

Prediction Equations

A line of fit is a line that closely approximates a set of data graphed in a scatter plot. The equation of a line of fit is called a prediction equation because it can be used to predict values not given in the data set.

To find a prediction equation for a set of data, select two points that seem to represent the data well. Then to write the prediction equation, use what you know about writing a linear equation when given two points on the line.

STORAGE COSTS

According to a certain prediction equation, the cost of 200 square feet of storage space is $60. The cost of 325 square feet of storage space is $160.

a. Find the slope of the prediction equation. What does it represent?

Since the cost depends upon the square footage, let \( x \) represent the amount of storage space in square feet and \( y \) represent the cost in dollars. The slope can be found using the formula \( \frac{y_2 - y_1}{x_2 - x_1} \). So, \( m = \frac{160 - 60}{325 - 200} = 0.8 \). The slope of the prediction equation is 0.8. This means that the price of storage increases 80¢ for each one-square-foot increase in storage space.

b. Find a prediction equation.

Using the slope and one of the points on the line, you can use the point-slope form to find a prediction equation.

\[ y - y_1 = m(x - x_1) \]

A prediction equation is \( y = 0.8x + 10 \).

SALARIES

The table below shows the years of experience for eight technicians at Lewis Techomatic and the hourly rate of pay each technician earns. Use the data a. Draw a scatter plot to show how years of experience are related to hourly rate of pay. Draw a line of fit. See graph.

b. Write a prediction equation to show how years of experience (\( x \)) and hourly rate of pay (\( y \)) are related. Sample answer using (1, 17) and (7, 34): \( y = 3.6x + 8.8 \).

Exercises

For Exercises 1–3, complete parts a–c for each set of data.

a. Draw a scatter plot.

b. Use two ordered pairs to write a prediction equation.

c. Use your prediction equation to predict the missing value.

1a.

1b. Sample answer using (1, 1) and (8, 15):

\[ y = 2x - 1 \]

1c. Sample answer: 19

2a.

2b. Sample answer using (5, 9) and (40, 44):

\[ y = \frac{4}{5}x + 1 \]

2c. Sample answer: 54

3a.

3b. Sample answer using (2, 16) and (7, 34):

\[ y = 3.6x + 8.8 \]

3c. Sample answer: 19.6

Chapter 2

NAME ______________________________ DATE ____________ PERIOD _____

2-5 Study Guide and Intervention

Modeling Real-World Data: Using Scatter Plots

Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.

Glencoe Algebra 2

Chapter 2

NAME ______________________________ DATE ____________ PERIOD _____

2-5 Study Guide and Intervention

Modeling Real-World Data: Using Scatter Plots

Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.

Glencoe Algebra 2

Chapter 2
Answers (Lesson 2-5)

1. AIRCRAFT
The table shows the maximum speed and altitude of different aircraft. Draw a scatter plot of this data.

2. TESTING
The scatter plot shows the height and test scores of students in a math class. Describe the correlation between heights and test scores.

3. STOCKS
The prices of a technology stock over 5 days are shown in the table. Draw a scatter plot of the data and a line of fit.

4. ALGAE
The scatter plot shows data recording the amount of algae and the temperature of the water in various aquarium tanks. Draw a line of fit for this data and write a prediction equation.

5. HEALTH
Alton has a treadmill that uses the time on the treadmill and the speed of walking or running to estimate the number of Calories he burns during a workout. The table gives workout times and Calories burned for several workouts.

6. SPORTS
For Exercises 5 and 6, use the scatter plot showing the height and the score of different contestants shooting darts.

5. What is the equation of the line of fit?
Sample answer: \( y = 0.3x + 2 \)

6. What do you predict someone 5 feet tall would score?
10 points

- **Table:**
  - **Time (min):** 18, 24, 30, 40, 42, 48, 52, 60
  - **Calories Burned:** 260, 280, 320, 380, 400, 440, 475, ?

- **Graph:**
  - X-axis: Time (min)
  - Y-axis: Calories Burned

- **Line of Fit:**
  - Equation: \( y = 0.3x + 2 \)

*Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.*
**Chapter 2**

**NAME ______________________________ DATE ____________ PERIOD _____

**2-6 Lesson Reading Guide**

Special Functions

**Get Ready for the Lesson**

Read the introduction to Lesson 2-6 in your textbook.

- What is the cost of mailing a letter that weighs 0.5 ounce? **$0.37 or 37 cents**
- Give three different weights of letters that would each cost 60 cents to mail. **Answers will vary. Sample answer: 1.1 ounces, 1.9 ounces, 2.0 ounces**

**Read the Lesson**

1. Find the value of each expression.
   - a. \(|-3| = 3\)
   - b. \(|\frac{6}{2}| = 6\)
   - c. \(|-4.01| = 4.01\)

2. Tell how the name of each kind of function can help you remember what the graph looks like.
   - a. **constant function** Sample answer: Something is constant if it does not change. The values of a constant function do not change, so the graph is a horizontal line.
   - b. **absolute value function** Sample answer: The absolute value of a number tells you how far it is from 0 on the number line. It makes no difference whether you go to the left or right so long as you go the same distance each time.
   - c. **step function** Sample answer: A step function's graph looks like steps that go up or down.
   - d. **identity function** Sample answer: The x- and y-values are always identical the same for any point on the graph. So the graph is a line through the origin that has slope 1.

**Remember What You Learned**

3. Many students find the greatest integer function confusing. Explain how you can use a number line to find the value of this function for any real number. **Answers will vary. Sample answer: Draw a number line that shows the integers. To find the value of the greatest integer function for any real number, place that number on the number line. If it is an integer, the value of the function is the number itself. If not, move to the integer directly to the left of the number you chose. This integer will give the value you need.**

**Answers (Lessons 2.5 and 2.6)**
### Special Functions

- **Constant Function**
  - Written as: \( f(x) = c \)
  - Graph: horizontal line

- **Identity Function**
  - Written as: \( f(x) = x \)
  - Graph: line through the origin with slope 1

- **Greatest Integer Function**
  - Written as: \( f(x) = \lfloor x \rfloor \)
  - Graph: one-unit horizontal segments, with right endpoints missing, arranged like steps

#### Step Functions, Constant Functions, and the Identity Function

The chart below lists some special functions you should be familiar with.

<table>
<thead>
<tr>
<th>Function</th>
<th>Written as</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>( f(x) = c )</td>
<td>horizontal line</td>
</tr>
<tr>
<td>Identity</td>
<td>( f(x) = x )</td>
<td>line through the origin with slope 1</td>
</tr>
<tr>
<td>Greatest Integer</td>
<td>( f(x) = \lfloor x \rfloor )</td>
<td>one-unit horizontal segments, with right endpoints missing, arranged like steps</td>
</tr>
</tbody>
</table>

The greatest integer function is an example of a step function, a function with a graph consisting of horizontal segments.

#### Exercises

1. Identify each function as a constant function, the identity function, a greatest integer function, or a step function.
   - \( f(x) = 3 \) is a constant function.
   - \( f(x) = x \) is the identity function.
   - \( f(x) = \lfloor x \rfloor \) is a greatest integer function.
   - \( f(x) = \) is a step function.

2. For each function, identify the domain and range:
   - \( f(x) = 2x \) for \( x \leq 2 \) has a domain of all real numbers and a range of \( \{y \mid y < 4\} \).
   - \( f(x) = \lfloor x \rfloor \) has a domain of all real numbers and a range of all integers.
   - \( f(x) = \) has a domain of all real numbers and a range such that all integers greater than or equal to the integer part of \( x \).

---

### Example

1. **Graph** \( f(x) = \) for \( x < 0 \). Graph the points and connect them. You would expect the graph to look similar to its parent function, \( f(x) = x \).
2. **Graph** \( f(x) = \lfloor x \rfloor \) for \( x < 2 \). Since 2 does not satisfy this inequality, stop with a circle at (2, 4). Next, graph the linear function \( f(x) = x \) for \( x \geq 2 \). Since 2 does satisfy this inequality, begin with a dot at (2, 1).

---

### Notes

- **Step Function**: A function with a graph that consists of horizontal segments.
- **Absolute Value Function**: Can be written as a piecewise function. A piecewise function is written using two or more expressions. Its graph is often disjointed.

---

**Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.**

**Example**

**Exercises**
Chapter 2

Skills Practice

2-6

Graph each function. Identify the domain and range.

1. \( f(x) = \frac{1}{3}x + \frac{2}{3} \)

2. \( f(x) = 3x - 2 \)

3. \( f(x) = 4x + 3 \)

4. \( f(x) = \begin{cases} x + 1 & \text{if } x < 0 \\ x - 1 & \text{if } x \geq 0 \end{cases} \)

5. \( f(x) = \begin{cases} |x| & \text{if } x \leq 0 \\ x^2 & \text{if } x > 0 \end{cases} \)

6. \( f(x) = \begin{cases} x^2 & \text{if } |x| < 1 \\ 2x & \text{if } |x| \geq 1 \end{cases} \)

7. \( f(x) = \begin{cases} 0 & \text{if } x < -1 \\ x & \text{if } -1 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases} \)

8. \( f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases} \)

9. \( f(x) = \begin{cases} 0 & \text{if } x < -1 \\ |x| & \text{if } -1 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases} \)

Answers

Lesson 2-6

Special Functions

Identify each function as S for step, C for constant, A for absolute value, or P for piecewise.

1. \( f(x) = \frac{1}{3}x + \frac{2}{3} \)

2. \( f(x) = 3x - 2 \)

3. \( f(x) = 4x + 3 \)

4. \( f(x) = \begin{cases} x + 1 & \text{if } x < 0 \\ x - 1 & \text{if } x \geq 0 \end{cases} \)

5. \( f(x) = \begin{cases} |x| & \text{if } x \leq 0 \\ x^2 & \text{if } x > 0 \end{cases} \)

6. \( f(x) = \begin{cases} x^2 & \text{if } |x| < 1 \\ 2x & \text{if } |x| \geq 1 \end{cases} \)

7. \( f(x) = \begin{cases} 0 & \text{if } x < -1 \\ x & \text{if } -1 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases} \)

8. \( f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases} \)

9. \( f(x) = \begin{cases} 0 & \text{if } x < -1 \\ |x| & \text{if } -1 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases} \)

10. \( f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases} \)

Graph each function. Identify the domain and range.

Business

A wholesaler charges a store $3.00 for each pound of candy and $0.50 for each pound of chocolate if there is less than 20 pounds of candy and $2.50 per pound for 20 or more pounds of candy. Draw a graph of the step function that represents this situation.
2-6 Word Problem Practice
Special Functions

1. SAVINGS Nathan puts $200 into a checking account as soon as he gets his paycheck. The value of his checking account is modeled by the formula \( \frac{200}{m} \), where \( m \) is the number of months that Nathan has been working. After 105 days, how much money is in the account? $600

2. FINANCE A financial advisor handles the transactions in a bank account. For every transaction, the advisor gets a 5% commission, regardless of whether the transaction is a deposit or withdrawal. Write a formula using the absolute value function for the advisor's commission. Let \( D \) represent the value of one transaction.

\[ C = 0.05 |D| \text{ or } C = |0.05D| \]

3. ROUNDEL A science teacher instructs students to round their measurements as follows: If a number is less than 0.5 of a millimeter, students are instructed to round down. If a number is exactly 0.5 or greater, students are told to round up to the next millimeter. Write a formula that takes a measurement \( x \) and yields the rounded off number. \( l = [x + 0.5] \text{ mm} \)

4. ARCHITECTURE The cross-section of a roof is shown in the figure. Write an absolute value function that models the shape of the roof.

\[ y = 6 - |x - 3| \]

5. Write a formula that gives the horizontal distance from the center of the dartboard. \( d = |x| \)

6. Write a formula using the greatest integer function that can be used to find the person's score. \( S = 3 - |x| \)

2-6 Enrichment
Graphing Greatest Integer Functions

Some equations involving the greatest integer function produce interesting graphs. It will be helpful to make a chart of values for each function and to use a colored pen or pencil.

Graph each function.
1. \( y = 2x - [x] \)
2. \( y = \frac{|x|}{[x]} \)
3. \( y = \frac{0.5x + 1}{[0.5x + 1]} \)
4. \( y = \frac{x}{[x]} \)
Graphing Inequalities

Read the introduction to Lesson 2-7 in your textbook.

1. Which of the combinations of passing yards and touchdown passes listed would Dana consider a good game?

2. Suppose that in one of the games Dana plays, Bledsoe passes 157 yards. What is the smallest number of touchdown passes he must get in order for Dana to consider this a good game?

3. When graphing a linear inequality in two variables, how do you know whether to make the boundary a solid line or a dashed line?

4. Describe some ways in which graphing an inequality in one variable on a number line is similar to graphing inequalities in a coordinate plane. How are they different?

5. Suppose that in one of the games Dana plays, Bledsoe passes 168 yards and 3 touchdowns. What is the line in the menu. When graphing the greatest integer function, it is important to remember what you learned.

6. When graphing the greatest integer function, it is important to remember what you learned.

Remember What You Learned

1. What do you know about graphing on a number line help you to graph inequalities in a coordinate plane? A boundary on a coordinate plane is similar to a circle on a number line: both are open and mean not included. A solid line is similar to a line on a number line: both are closed and mean included. A boundary on a coordinate plane is similar to a circle on a number line: both are open and mean not included. A solid line is similar to a line on a number line: both are closed and mean included.

Exercises

1. The first one:

2. The second one:

3. The third one:

4. The fourth one:

5. The fifth one:

6. The sixth one:

7. The seventh one:

8. The eighth one:

9. The ninth one:

10. The tenth one:

Graph each function. Evaluate it for x = 1, 1.1, 2, 3, 4, 6. Compare the graph of the function to the graph of f(x) = |x|.

Graph each function. Evaluate it for x = 1, 1.1, 2, 3, 4, 6. Compare the graph of the function to the graph of f(x) = |x|.

Graph each function. Evaluate it for x = 1, 1.1, 2, 3, 4, 6. Compare the graph of the function to the graph of f(x) = |x|.

Graph each function. Evaluate it for x = 1, 1.1, 2, 3, 4, 6. Compare the graph of the function to the graph of f(x) = |x|.
Graphing Linear Inequalities

1. Graph the boundary; that is, the related linear equation. If the inequality symbol is \( \leq \) or \( \geq \), the boundary is solid. If the inequality symbol is \( < \) or \( > \), the boundary is dashed.

2. Choose a point not on the boundary and test it in the inequality. (0, 0) is a good point to choose if the boundary does not pass through the origin.

3. If a true inequality results, shade the half-plane containing your test point. If a false inequality results, shade the other half-plane.

Example

Graph \( y \geq 2x - 1 \).

The boundary is the graph of \( y = 2x - 1 \).

Use the slope-intercept form, \( y = mx + b \), to graph the boundary line.

The boundary line should be solid.

Now test the point \((0, 0)\).

\[ 0 \geq 2(0) - 1 = 0 \]

Since \( 0 \geq 0 \), the graph of the boundary is solid.

Shade the region that contains \((0, 0)\).

Example

Graph \( y \leq 3|\frac{x}{2} - 1| \).

First graph the equation \( y = 3|\frac{x}{2} - 1| \).

Since the inequality is \( \leq \), the graph of the boundary is solid.

Test \((0, 0)\).

\[ 0 \leq 3|0 - 1| = 3 \]

\[ 0 \leq 3 \] is true

Shade the region that contains \((0, 0)\).

Exercises

Graph each inequality.

1. \( y < 3x + 1 \)
2. \( y \geq x - 5 \)
3. \( 4x + y \leq -1 \)
4. \( y < \frac{x}{2} - 4 \)
5. \( x + y > 6 \)
6. \( 0.5x - 0.25y < 1.5 \)
7. \( 2 - x + y > -1 \)
8. \( y < 3|x| - 3 \)
9. \( y \leq |1 - x| + 4 \)
Answers (Lesson 2-7)

Graph each inequality.

1. \( y \leq -4 \)
2. \( x > 2 \)
3. \( x \leq -1 \)
4. \( y < \frac{1}{2}x + 3 \)
5. \( y > |x| - 1 \)
6. \( x - 3y \leq 6 \)
7. \( x = 3 \)
8. \( y = x - 1 \)

For Exercises 10-12, use the following information.

A school system is buying new computers. They will buy desktop computers costing $1000 per unit, and notebook computers costing $1200 per unit. The total cost of the computers cannot exceed $80,000.

10. Write an inequality that describes this situation.
   \[ 1000d + 1200n \leq 80,000 \]

11. Graph the inequality.

12. If the school wants to buy 50 of the desktop computers and 25 of the notebook computers, will they have enough money? **Yes**
2.7 Word Problem Practice

Graphing Inequalities

1. FRAMES The dimensions of a rectangular frame that can be made from a 50 inch plank of wood are limited by the inequality \( l + w \leq 25 \). Graph this inequality.

2. BUILDING CODE A city has a building code that limits the height of buildings around the central park. The code says that all buildings must be less than 0.1 \( x \) in height where \( x \) is the distance of the building from the center of the park. Assume that the park center is located at 0. Graph the inequality that represents the building code.

3. LIVESTOCK During the winter, a horse requires about 36 liters of water per day and a sheep requires about 3.6 liters per day. A farmer is able to supply his horses and sheep with a total of 300 liters of water each day. Write an inequality that represents the possible number of horses and sheep this farmer can keep. \( 36h + 3.6s \leq 300 \)

4. WEIGHT A delivery crew is going to load a truck with tables and chairs. The truck's weight limitations are represented by the inequality \( 200t + 60c < 1200 \), where \( t \) is the number of tables and \( c \) is the number of chairs. Graph this inequality.

ART For Exercises 5–7, use the following information.

An artist can sell each drawing for $100 and each painting for $400. He hopes to make at least $2000 every month.

5. Write an inequality that expresses how many paintings and/or drawings the artist needs to sell each month to reach his goal. \( 100p + 400d \geq 2000 \)

6. Graph the inequality.

7. If David sells three paintings one month, how many drawings would he have to sell in the same month to reach $2000? 8

2.7 Enrichment

Limits

The concept of the limit is central to many areas of mathematics, especially to calculus. For example, consider the expression, \( 3x + 2 \). As the value of \( x \) approaches 1, the value of the expression approaches 5, as shown in the table below. The process of systematically choosing values closer to 1 and producing values closer to 5 demonstrates finding the limit of the expression.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 3x + 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>4.979</td>
</tr>
<tr>
<td>0.999</td>
<td>4.997</td>
</tr>
<tr>
<td>0.9999</td>
<td>4.9997</td>
</tr>
</tbody>
</table>

Find the limits for each expression as \( x \) approaches the value given.

1. \( 2x + 2 \) as \( x \) approaches 5

2. \( x - 5 \) as \( x \) approaches 11

3. \( \frac{3x + 5}{x - 6} \) as \( x \) approaches 1

4. \( \frac{5x - 2}{x + 1} \) as \( x \) approaches 1

5. \( \frac{3x + 5}{x - 6} \) as \( x \) approaches 100

6. \( \frac{5x - 2}{x + 1} \) as \( x \) approaches 100

7. \( \frac{3x + 5}{x - 6} \) as \( x \) approaches 1000

8. \( \frac{5x - 2}{x + 1} \) as \( x \) approaches 1000

9. What do you notice about the limits you found in exercises 3–8? In the first set, the limit is approaching 3 as \( x \) increases. In the second set, the limit is approaching 5 as \( x \) increases.