**Anticipation Guide**

**Radical Equations**

**STEP 1**
Before you begin Chapter 7

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

<table>
<thead>
<tr>
<th>Statement</th>
<th>STEP 2 A or D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Functions can be added or subtracted in the same way as polynomials.</td>
<td>A</td>
</tr>
<tr>
<td>2. A composition of functions, ( f(g(x)) ), is found by multiplying ( f(x) ) by ( g(x) ).</td>
<td>D</td>
</tr>
<tr>
<td>3. The inverse of a function is the set of ordered pairs obtained by taking the opposite of each coordinate in the original ordered pairs.</td>
<td>D</td>
</tr>
<tr>
<td>4. Two functions are inverses of each other only if their compositions are the identity function.</td>
<td>A</td>
</tr>
<tr>
<td>5. The domain of ( y = \sqrt{x - 3} ) would be ( x \geq 3 ).</td>
<td>A</td>
</tr>
<tr>
<td>6. The principal root of any ( n )th root is always positive.</td>
<td>D</td>
</tr>
<tr>
<td>7. The radical expression ( \sqrt[3]{a} ) is in simplest form.</td>
<td>D</td>
</tr>
<tr>
<td>8. ( 4 + \sqrt{3} ) and ( 4 - \sqrt{3} ) are conjugates of each other.</td>
<td>A</td>
</tr>
<tr>
<td>9. ( \sqrt{a} ) is the same as ( \sqrt[3]{a} ).</td>
<td>D</td>
</tr>
<tr>
<td>10. To solve an equation containing the square root of the variable, square both sides of the equation.</td>
<td>A</td>
</tr>
</tbody>
</table>

**STEP 2**
After you complete Chapter 7

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

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**Lesson Reading Guide**

**Operations on Functions**

Get Ready for the Lesson

Read the introduction to Lesson 7-1 in your textbook.

Describe two ways to calculate Ms. Coffman's profit from the sale of 50 birdhouses. (Do not actually calculate her profit.) Sample answer: 1. Find the revenue by substituting 50 for \( x \) in the expression 125\( x \). Next, find the cost by substituting 50 for \( x \) in the expression 65\( x \) + 5400. Finally, subtract the cost from the revenue to find the profit.

2. Form the profit function \( p(x) = r(x) - c(x) = 125x - (65x + 5400) = 60x - 5400 \). Substitute 50 for \( x \) in the expression 60\( x \) - 5400.

Read the Lesson

1. Determine whether each statement is true or false. (Remember that true means always true.)
   - a. If \( f \) and \( g \) are polynomial functions, then \( f + g \) is a polynomial function. true
   - b. If \( f \) and \( g \) are polynomial functions, then \( f^2 \) is a polynomial function. false
   - c. If \( f \) and \( g \) are polynomial functions, the domain of the function \( f \cdot g \) is the set of all real numbers. true
   - d. If \( f(x) = 3x + 2 \) and \( g(x) = x - 4 \), the domain of the function \( f \cdot g \) is the set of all real numbers. false
   - e. If \( f \) and \( g \) are polynomial functions, then \( (f + g)(x) = (g + f)(x) \). false
   - f. If \( f \) and \( g \) are polynomial functions, then \( (f - g)(x) = g \cdot f(x) \). true

2. Let \( f(x) = 2x - 5 \) and \( g(x) = x^2 + 1 \).
   - a. Explain in words how you would find \( (f + g)(-3) \). (Do not actually do any calculations.) Sample answer: Square \(-3\) and add 1. Take the number you get, multiply it by 2, and subtract 5.
   - b. Explain in words how you would find \( (g - f)(-3) \). (Do not actually do any calculations.) Sample answer: Multiply \(-3\) by 2 and subtract 5. Take the number you get, square it, and add 1.

Remember What You Learned

3. Some students have trouble remembering the correct order in which to apply the two original functions when evaluating a composite function. Write three sentences, each of which explains how to do this in a slightly different way. (Hint: Use the word closest in the first sentence, the words inside and outside in the second, and the words left and right in the third.) Sample answer: 1. The function that is written closest to the variable is applied first. 2. Work from the inside to the outside. 3. Work from right to left.
7-1 Study Guide and Intervention

Operations on Functions

Arithmetic Operations

Composition of Functions

Example 1

For \( f = \{ (1, 2), (3, 3), (2, 4), (4, 1) \} \) and \( g = \{ (1, 3), (3, 4), (2, 2), (4, 1) \} \), find \( f \circ g \) and \( g \circ f \) if they exist.

\[
\begin{align*}
(\circ)\quad & f \circ g = \{ (1, 3), (3, 9), (2, 2), (4, 1) \} \\
& g \circ f = \{ (1, 8), (9, 8), (2, 9), (1, 6) \}
\end{align*}
\]

Example 2

Find \([h \circ f(x)]\) and \([g \circ h(x)]\) for \( g(x) = 3x - 4 \) and \( h(x) = x^2 - 1 \).

\[
\begin{align*}
[g \circ h(x)] &= g(h(x)) = 3x^2 - 4 \\
&= \{ (3, 6), (9, 2), (15, 6) \}
\end{align*}
\]

Exercises

For each set of ordered pairs, find \( f \circ g \) and \( g \circ f \) if they exist.

1. \( f = \{ (-1, 2), (3, 6), (0, 9) \} \)
   \( g = \{ (1, 3), (2, 2), (4, 1) \} \)

2. \( f = \{ 15, 20, 0, 4, -3, 4, 0, 4 \} \)
   \( g = \{ (3, 7), (-2, 6), (4, 2), (4, 10) \} \)

Find \( (f \circ g)(x) \) and \( (g \circ f)(x) \).

3. \( f(x) = 2x + 2; g(x) = \frac{5x - 1}{x + 1} \)

4. \( f(x) = x^2 + 1; g(x) = x^2 - 4x + 4 \)

5. \( f(x) = x^2 + 2x; g(x) = 3 - x \)

6. \( f(x) = x^2 + x + 1; g(x) = 3 - x \)

\[
\begin{align*}
(f \circ g)(x) &= 10x + 5 \\
&= \{ (1, 15), (2, 21), (3, 27) \}
\end{align*}
\]

\[
\begin{align*}
(g \circ f)(x) &= -10x + 36 \\
&= \{ (1, -41), (2, -64), (3, -87) \}
\end{align*}
\]
7-1 Skills Practice

Operations on Functions

Find \((f + g)(x)\), \((f - g)(x)\), \((f \cdot g)(x)\), and \(\left(\frac{f}{g}\right)(x)\) for each \(f(x)\) and \(g(x)\).

1. \(f(x) = 3x^2 + 2x + 1; g(x) = x - 4\)
   \[
   (f + g)(x) = 3x^2 + 3x + 1 \\
   (f - g)(x) = 3x^2 + 3x + 1 \\
   (f \cdot g)(x) = 3x^5 + 6x^4 + 5x^3 \\
   \left(\frac{f}{g}\right)(x) = \frac{3x^2}{x^2} + \frac{3x}{x} + \frac{1}{x}
   \]

2. \(f(x) = x^3 + 3x + 2; g(x) = x^2 - 5 \)
   \[
   (f + g)(x) = x^5 + 3x^3 + 3x + 2 \\
   (f - g)(x) = x^3 + 3x + 2 \\
   (f \cdot g)(x) = x^5 + 3x^3 + 3x + 2 \\
   \left(\frac{f}{g}\right)(x) = \frac{x^3 + 3x + 2}{x^2 - 5}
   \]

For each set of ordered pairs, find \(f \cdot g\) and \(g \cdot f\) if they exist.

5. \(f(x) = 0, 0, 4, \) \(g(x) = 0, 0, 4, 0\)
   \[
   f \cdot g = (0, 0) \text{ and } g \cdot f = (0, 0)
   \]

6. \(f(x) = 0, 1, 3; \) \(g(x) = 0, 3, 1\)
   \[
   f \cdot g = (0, 0) \text{ and } g \cdot f = (0, 0)
   \]

7. \(f(x) = -4, 3, 1; \) \(g(x) = 1, -4, 3, 1\)
   \[
   f \cdot g = (1, -4, 3, 1) \text{ and } g \cdot f = (1, -4, 3, 1)
   \]

Find \(g \cdot h(x)\) and \(h \cdot g(x)\).

9. \(f(x) = 2x + 4; g(x) = 2x^2 + 2h(x) = x + 2\)

10. \(f(x) = -3x^2 - 12x + 3; h(x) = x^2 - 4\)

11. \(f(x) = -6x + 6; h(x) = x^2 - 4\)

12. \(f(x) = -3x^2 - 12x + 3; h(x) = x^2 - 4\)

13. \(f(x) = 5x^2 + 5x - 5; h(x) = x^2 - 4\)

14. \(f(x) = x^2 + 2x^2 - 1; h(x) = 2x^2 - 3\)

If \(f(x) = 3x\), \(g(x) = x + 4\), and \(h(x) = x^2 - 1\), find each value.

15. \(f(\bar{g}(1)) \) 15

16. \(g(\bar{h}(0)) \) 3

17. \(\bar{g}(\bar{f}(1)) \) 1

18. \(h(f(5)) \) 224

19. \(g(h(-3)) \) 12

20. \(h(f(10)) \) 899

21. \(f(h(\bar{g}(1))) \) 189

22. \(f(\bar{h} + \bar{g})(1)) \) 72

23. \(f(\bar{g} + \bar{h})(1)) \) 21

Chapter 7

Answers

7-1 Practice

Operations on Functions

Find \((f + g)(x)\), \((f - g)(x)\), \((f \cdot g)(x)\), and \(\left(\frac{f}{g}\right)(x)\) for each \(f(x)\) and \(g(x)\).

1. \(f(x) = 2x + 1; \) \(g(x) = x - 4\)
   \[
   (f + g)(x) = 3x - 3 \\
   (f - g)(x) = 3x - 3 \\
   (f \cdot g)(x) = 2x^2 - 3x - 12 \\
   \left(\frac{f}{g}\right)(x) = \frac{2x + 1}{x - 4}
   \]

2. \(f(x) = -x^2 + 3x + 2; \) \(g(x) = x + 2\)
   \[
   (f + g)(x) = -x^2 + 4x + 4 \\
   (f - g)(x) = -x^2 + 4x + 4 \\
   (f \cdot g)(x) = -x^4 + x^2 + 4x^2 - 3x - 8 \\
   \left(\frac{f}{g}\right)(x) = \frac{-x^2 + 3x + 2}{x + 2}
   \]

For each set of ordered pairs, find \(f \cdot g\) and \(g \cdot f\) if they exist.

4. \(f(x) = (0, -9, 1); \) \(g(x) = (0, 3, 3)\)

5. \(f(x) = (0, -3, 2); \) \(g(x) = (0, 3, -2)\)

6. \(f(x) = (0, 4, 0); \) \(g(x) = (0, 2, 0)\)

7. \(f(x) = (0, 0, 3); \) \(g(x) = (0, 0, 2)\)

Find \(g \cdot h(x)\) and \(h \cdot g(x)\).

8. \(f(x) = 3x; \) \(g(x) = -3x^2 + 2x + 1\)

9. \(f(x) = -x^3 + 3x - 1; \) \(g(x) = x^3 - 1\)

10. \(f(x) = x + 6; \) \(g(x) = 3x^2 + 2x + 1\)

If \(f(x) = x^2, g(x) = 3x, \) and \(h(x) = x + 4\), find each value.

11. \(f(\bar{g}(1)) \) 25

12. \(f(\bar{h})(-2) \) 10

13. \(f(\bar{h})(2) \) 25

14. \(f(\bar{h})(-1) \) 11

15. \(f(\bar{h})(10) \) 320

16. \(f(\bar{h})(1) \) 1600

23. BUSINESS The function \(f(x) = 1000 - 0.01x^2\) models the manufacturing cost per item when \(x\) items are produced, and \(g(x) = 150 - 0.001x^2\) models the service cost per item. Write a function \(C(x)\) for the total manufacturing and service cost per item.

24. MEASUREMENT The formula \(f = \frac{1}{4}t\) converts inches to feet, \(f\), and \(m = \frac{1}{12000}\) converts feet to miles, \(m\). Write a composition of functions that converts inches to miles.

\[
\text{[m = f(n)] = 0.016360} \]

Chapter 7

Glencoe Algebra 2
1. **AREA** A painter wants to know the area of a figure made by joining an equilateral triangle and square along an edge. The function \( f(s) = \frac{\sqrt{3}}{4} s^2 \) gives the area of an equilateral triangle with side \( s \). The function \( g(s) = s^2 \) gives the area of a square with side \( s \). What function \( h(s) \) gives the area of the figure as a function of its side length \( s \)?

\[ h(s) = (f + g)(s) = \left( \frac{\sqrt{3}}{4} + 1 \right) s^2 \]

2. **PRICING** A computer company decides to continuously adjust the pricing of and discounts to its products to an effort to remain competitive. The function \( P(t) \) gives the sale price of its Super2000 computer as a function of time. The function \( D(t) \) gives the value of a special discount it offers to valued customers. How much would valued customers have to pay for one Super2000 computer? \( (P - D)(t) \)

3. **LAVA** A freshly ejected lava rock immediately begins to cool down. The temperature of the lava rock in degrees Fahrenheit as a function of time is given by \( T(t) \). Let \( C(T) \) be the function that gives degrees Celsius as a function of degrees Fahrenheit. What function gives the temperature of the lava rock in degrees Celsius as a function of time? \( C(T(t)) \)

4. **ENGINEERING** A group of engineers is designing a staple gun. One team determines that the speed of impact \( s \) of the staple (in feet per second) as a function of the handle length \( f \) (in inches) is given by \( s(f) = 40 + 3f \). A second team determines that the number of sheets \( N \) that can be stapled as a function of the impact speed is given by \( N(s) = \frac{10}{s} \). What function gives \( N \) as a function of \( f \)?

\[ N(s(f)) = 10 + f \]

5. What function describes the air temperature Hannah and Terry felt at different times during their trip? \( T(A(t)) \)

6. Sketch a graph of the function you wrote for Exercise 5 based on the graphs for \( T(A) \) and \( A(t) \) that are given.
Lesson Reading Guide

Inverse Functions and Relations

Get Ready for the Lesson

Read the introduction to Lesson 7-2 in your textbook.

A function multiplies a number by 3 and then adds 5 to the result. What does the inverse function do, and in what order? Sample answer: It first subtracts 5 from the number and then divides the result by 3.

Read the Lesson

1. Complete each statement.
   a. If two relations are inverses, the domain of one relation is the _____ range _____ of the other.
   b. Suppose that \( g \) is a relation and that the point \((4, -2)\) is on its graph. Then a point on the graph of \( g^{-1} \) is _____ \(-2, 4\) _____.
   c. The horizontal line test can be used on the graph of a function to determine whether the function has an inverse function.
   d. If you are given the graph of a function, you can find the graph of its inverse by reflecting the original graph over the line with equation \( y = x \).
   e. If \( f \) and \( g \) are inverse functions, then \((f \circ g)(x) = x \) and \((g \circ f)(x) = x \).
   f. A function has an inverse that is also a function only if the given function is _____ one-to-one _____.
   g. Suppose that \( h(x) \) is a function whose inverse is also a function. If \( h(5) = 12 \), then \( h^{-1}(12) = _____ 5 _____ \).

2. Assume that \( f(x) \) is a one-to-one function defined by an algebraic equation. Write the four steps you would follow in order to find the equation for \( f^{-1}(x) \).
   1. Replace \( f(x) \) with \( y \) in the original equation.
   2. Interchange \( x \) and \( y \).
   3. Solve for \( y \).
   4. Replace \( y \) with \( f^{-1}(x) \).

Remember What You Learned

3. A good way to remember something new is to relate it to something you already know. How are the vertical and horizontal line tests related? Sample answer: The vertical line test determines whether a relation is a function because the ordered pairs in a function cannot have any repeated \( x \)-values. The horizontal line test determines whether a function is one-to-one because a one-to-one function cannot have any repeated \( y \)-values.

Exercises

1. Find \((h \circ g)(x) = 3x + 2\). How does it compare to \( h(x) \)?
   \( h(x) = 3x^2 - 2x \), and \( h(x) = 3x^2 + x + 2 \). Does it appear that \( f(x) + g(x) = h(x) \)?

2. Change the functions in the spreadsheet to \( f(x) = \frac{x}{3} \), \( g(x) = 1 - x^2 \), and \( h(x) = 1 + \frac{5}{2} - x^2 \). How are these functions related? Is it true that \( f(x) + g(x) = h(x) \)? Yes.

3. Make a conjecture about \( f(x) + g(x) \) for any functions \( f(x) \) and \( g(x) \).
   \((f + g)(x) = f(x) + g(x)\)

4. Make a conjecture about \( f(x) - g(x) \) for any functions \( f(x) \) and \( g(x) \). Use the spreadsheet to test your conjecture. Does it appear to be true? Explain your answer. \((f - g)(x) = f(x) - g(x); See students' work.\)

Find \((f \circ g)(x), (g \circ f)(x), (f - g)(x), \) and \( g(x) \) for each \( f(x) \) and \( g(x) \). Use the spreadsheet to find function values to verify your solutions. 7-7. See students' spreadsheets.

\begin{align*}
5. f(x) &= 6x + 8 \\
g(x) &= 9x - 9 + x \\
7x + 17; 5x - 1
\end{align*}

\begin{align*}
6. f(x) &= x^2 + 1 \\
g(x) &= 3x - 4 \\
x^2 + 3x - 3; x^2 - 3x + 5
\end{align*}

\begin{align*}
7. f(x) &= 10x \\
g(x) &= 6 - x^2 \\
x^2 + 6; 11x^2 - 6
\end{align*}
**Inverse Relations**

Two relations are inverse relations if and only if whenever one relation contains the element \((a, b)\), the other relation contains the element \((b, a)\).

**Property of Inverse Functions**

Suppose \(f\) and \(f^{-1}\) are inverse functions. Then \(f(g(x)) = x\) if and only if \(f^{-1}(b) = a\).

**Example**

Find the inverse of the function \(f(x) = \frac{2}{3}x - \frac{1}{3}\). Then graph the function and its inverse.

**Step 1** Replace \(f(x)\) with \(y\) in the original equation.

\[ x = \frac{2}{3}y - \frac{1}{3} \]

**Step 2** Interchange \(x\) and \(y\).

\[ y = \frac{2}{3}x - \frac{1}{3} \]

**Step 3** Solve for \(y\).

\[ 3y = 2x - 1 \]

\[ y = \frac{2}{3}x - \frac{1}{3} \]

The inverse of \(f(x) = \frac{2}{3}x - \frac{1}{3}\) is \(f^{-1}(x) = \frac{2}{3}x + 1\).

**Exercises**

Find the inverse of each function. Then graph the function and its inverse.

1. \(f(x) = \frac{3}{2}x - 1\)
   \(f^{-1}(x) = \frac{2}{3}x + 3\)

2. \(f(x) = 2x - 3\)
   \(f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}\)

3. \(f(x) = \frac{1}{2}x - 2\)
   \(f^{-1}(x) = 4x + 8\)

**Example 1**

Determine whether \(f(x) = 2x - 7\) and \(g(x) = \frac{1}{3}(x + 7)\) are inverse functions.

\[ f(g(x)) = f\left(\frac{1}{3}(x + 7)\right) = \frac{3}{2}x - 7 \]

\[ g(f(x)) = g\left(\frac{1}{3}x - 7\right) = x + 7 - 7 = x \]

The functions are inverses since both \(f(g(x)) = x\) and \(g(f(x)) = x\).

**Example 2**

Determine whether \(f(x) = 4x + \frac{1}{2}\) and \(g(x) = \frac{4}{3}x - 3\) are inverse functions.

\[ f(g(x)) = f\left(\frac{4}{3}x - 3\right) = \frac{3}{2}x - 3 \]

\[ g(f(x)) = g\left(\frac{4}{3}x - \frac{1}{2}\right) = x - 12 + \frac{1}{2} \]

Since \(f(g(x)) \neq x\), the functions are not inverses.
### 7-2 Skills Practice

**Inverse Functions and Relations**

Find the inverse of each relation.

1. $(3, 1), (4, -3), (8, -3)$
   \[ (1, 3), (5, 4), (3, -8) \]
2. $(-7, 1), (0, 5), (5, -1)$
   \[ (1, -7), (5, 0), (-1, 5) \]
3. $(-10, -2), (-7, 6), (-4, -2), (-4, 0)$
   \[ (2, -10), (6, -7), (-2, -4), (0, -4) \]
4. $(0, -9), (5, -3), (6, 6), (8, -3)$
   \[ (-9, 0), (-3, 5), (6, 6), (-3, 8) \]
5. $(-4, 12), (0, 7), (9, -1), (10, -5)$
   \[ (12, -4), (7, 0), (-1, 9), (-5, 10) \]
6. $(-4, 1), (-4, 3), (0, -8), (8, -9)$
   \[ (1, -4), (3, -4), (-8, 0), (-9, 8) \]

Find the inverse of each function. Then graph the function and its inverse.

7. $y = 4$
   \[ x = 4 \]
   \[ f^{-1}(x) = \frac{1}{2}x \]
   \[ f^{-1}(x) = x - 2 \]

8. $f(x) = 3x$
   \[ g^{-1}(x) = \frac{1}{3}x \]
   \[ g^{-1}(x) = x - 1 \]

9. $f(x) = x + 2$
   \[ g^{-1}(x) = \frac{1}{3}x + 2 \]
   \[ g^{-1}(x) = x - 3 \]

10. $g(x) = 2x - 1$
    \[ g^{-1}(x) = \frac{x + 1}{2} \]

11. $h(x) = \frac{1}{4}x$
    \[ h^{-1}(x) = 4x \]

12. $y = \frac{2}{3}x + 2$
    \[ y = \frac{3}{2}x - 3 \]

Determine whether each pair of functions are inverse functions.

13. $f(x) = x - 1$
    \[ g(x) = 1 - x \]
    Yes

14. $f(x) = 2x + 3$
    \[ g(x) = \frac{1}{2}(x - 3) \]
    Yes

15. $f(x) = 5x - 5$
    \[ g(x) = \frac{1}{5}x + 1 \]
    Yes

16. $f(x) = 2x$
    \[ g(x) = \frac{1}{2}x \]
    No

17. $h(x) = 6x - 2$
    \[ g(x) = \frac{1}{6}x + 3 \]
    No

18. $f(x) = 8x - 10$
    \[ g(x) = \frac{3}{8}x + 5 \]
    Yes

### 7-2 Practice

**Inverse Functions and Relations**

Find the inverse of each relation.

1. $(10, 3), (4, 2), (5, -6)$
   \[ (3, 10), (2, 4), (-6, 5) \]
2. $(-5, 1), (-5, -1), (-5, 8)$
   \[ (1, -5), (-5, 1), (-5, 8) \]
3. $(-3, -7), (0, -1), (5, 9), (7, 13)$
   \[ (7, -3), (-7, 0), (1, 5), (3, 7) \]
4. $(8, -2), (10, 5), (12, 6), (14, 7)$
   \[ (-2, 8), (5, 10), (6, 12), (7, 14) \]
5. $(5, -4), (1, 2), (3, 4), (7, 8)$
   \[ (-4, 5), (2, 1), (4, 3), (8, 7) \]
6. $(-3, -9), (-2, 4), (0, 0), (1, 1)$
   \[ (9, -3), (4, -2), (0, 0), (1, 1) \]

Find the inverse of each function. Then graph the function and its inverse.

7. $f(x) = \frac{3}{4}x$
   \[ f^{-1}(x) = \frac{4}{3}x \]
   \[ g^{-1}(x) = x - 3 \]
   \[ y = \frac{3}{2}x - 2 \]

8. $g(x) = 3 + x$
   \[ g^{-1}(x) = 3 - x \]

9. $y = 3x - 2$
   \[ f^{-1}(x) = \frac{2}{3}x \]

Determine whether each pair of functions are inverse functions.

10. $f(x) = x + 6$
    Yes
    \[ g(x) = x - 6 \]
    \[ g(x) = \frac{1}{3}x - 1 \]

11. $f(x) = -4x + 1$
    Yes
    \[ g(x) = \frac{1}{4}(1 - x) \]
    \[ g(x) = \frac{1}{13}x - 1 \]

12. $g(x) = 13x - 13$
    No
    \[ h(x) = \frac{1}{3}x - 1 \]
    \[ h(x) = \frac{1}{2}x - 4 \]

13. $f(x) = 2x$
    No
    \[ g(x) = \frac{1}{2}x \]
    \[ g(x) = \frac{1}{6}x \]

14. $f(x) = \frac{6}{7}x$
    Yes
    \[ g(x) = \frac{7}{6}x \]
    \[ g(x) = \frac{1}{6}x - 4 \]

15. $g(x) = -2x$
    Yes
    \[ h(x) = \frac{1}{2}x \]
    \[ h(x) = \frac{1}{6}x \]

16. **MEASUREMENT**
    The points (63, 121), (71, 180), (67, 140), (65, 108), and (72, 165) give the weight in pounds as a function of height in inches for 5 students in a class. Give the points for these students that represent height as a function of weight.
    \[ (121, 63), (180, 71), (140, 67), (108, 65), (165, 72) \]

17. **MODELING**
    For Exercises 17 and 18, use the following information.
    The Clearays are replacing the flooring in their 15 foot by 18 foot kitchen. The new flooring costs $17.99 per square yard. The formula $f(x) = 9x$ converts square yards to square feet.

    Find the inverse $f^{-1}(x)$.
    What is the significance of $f^{-1}(x)$ for the Clearays?
    $f^{-1}(x) = \frac{1}{9}x$
    It will allow them to convert the square footage of their kitchen floor to square yards, so they can then calculate the cost of the new flooring.

    **What will the new flooring cost the Clearays?**
    $\$359.70$
The order in which the operation and 1 is given for the set of integers. What is the identity element for the operation of subtraction under an operation? Is subtraction a noncommutative operation for the set of integers? Write an informal definition of noncommutative. The order in which the elements are used with the operation can affect the result.

1. VOLUME Jason wants to make a spherical water cooler that can hold half a cubic meter of water. He knows that 

   \[ V = \frac{4}{3} \pi r^3, \]

   but he needs to know how to find \( r \) given \( V \). Find this inverse function.

   \[ r = \frac{3\sqrt[3]{V}}{4\pi} \]

2. EXERCISE Alex began a new exercise routine. To gain the maximum benefit from his exercise, Alex calculated his maximum target heart rate using the function, \( f(x) = 0.85(220 - x) \) where \( x \) represents his age. Find the inverse of this function.

   \[ f^{-1}(x) = 220 - \frac{x}{0.85} \]

3. ROCKETS The altitude of a rocket in feet as a function of time is given by \( f(t) = 49t^2 \), where \( t \geq 0 \). Find the inverse of this function and determine the times when the rocket will be 10, 100, and 1000 feet high. Round your answers to the nearest hundredth of a second.

   \[ f^{-1}(t) = \frac{\sqrt{t}}{7}; \ 10 \text{ ft at } 0.45 \text{ s,} \]

   \[ 100 \text{ ft at } 1.43 \text{ s,} \ 1000 \text{ ft at } 4.52 \text{ s} \]

4. SELF-INVERTIBLE Karen finds the incomplete graph of a function in the back of her engineering handbook. The function is graphed in the figure below.

   Karen knows that this function is its own inverse. Armed with this knowledge, extend the graph for values of \( x \) between -7 and 2.

PLANETS For Exercises 5 and 6, use the following information.

The approximate distance of a planet from the Sun is given by \( d = T^1 \) where \( d \) is distance in astronomical units and \( T \) is Earth years. An astronomical unit is the distance of the Earth from the Sun.

5. Solve for \( T \) in terms of \( d \).

   \[ T = d^{1.5} \]

6. Pluto is about 39.44 times as far from the Sun as the Earth. About how many years does it take Pluto to orbit the Sun?

   \[ 248 \text{ yr} \]

Reading Algebra

In mathematics, the term group has a special meaning. The following numbered sentences discuss the idea of group and one interesting example of a group.

1. To be a group, a set of elements and a binary operation must satisfy four conditions: the set must be closed under the operation, the operation must be associative, there must be an identity element, and every element must have an inverse.

2. The following six functions form a group under the operation of composition of functions:

   \[ f_1(x) = x, \ f_2(x) = \frac{1}{x}, \ f_3(x) = 1 - x, \]

   \[ f_4(x) = \frac{(x-1)}{x}, \ f_5(x) = \frac{x}{(x-1)}, \text{ and } f_6(x) = \frac{1}{(1-x)}. \]

3. This group is an example of a noncommutative group. For example, \( f_1 \circ f_2 = f_6, \) but \( f_2 \circ f_1 = f_4. \)

4. Some experimentation with this group will show that the identity element is \( f_1. \)

5. Every element is its own inverse except for \( f_4 \) and \( f_6, \) each of which is the inverse of the other.

Use the paragraph to answer these questions.

1. Explain what it means to say that a set is closed under an operation. Is the set of positive integers closed under subtraction? Performing the operation on any two elements of the set results in an element of the same set. \( 3 \) and 4 are positive integers but \( 3 - 4 \) is not.

2. Subtraction is a noncommutative operation for the set of integers. Write an informal definition of noncommutative. The order in which the elements are used with the operation can affect the result.

3. For the set of integers, what is the identity element for the operation of multiplication? Justify your answer.

   1. because, for every integer \( a, a \cdot 1 = a \) and \( 1 \cdot a = a. \)

4. Explain how the following statement relates to sentence 05:

   \[ (f_6 \circ f_1)(x) = f_6[f_1(x)] = f_6\left(\frac{1}{(1-x)}\right) = \frac{1}{1-x} = x = f_1(x). \]

   It shows that \( f_4 \) is the inverse of \( f_6. \)
Answers (Lesson 7-3)

Square Root Functions

Graph each function. State the domain and range of the function.

1. \( y = \sqrt{x} \)
   - Domain: \( x \geq 0 \)
   - Range: \( y \geq 0 \)

2. \( y = -\sqrt{x} \)
   - Domain: \( x \geq 0 \)
   - Range: \( y \leq 0 \)

3. \( y = \sqrt{x} - 3 \)
   - Domain: \( x \geq 0 \)
   - Range: \( y \geq -3 \)

4. \( y = \sqrt{x} + 3 \)
   - Domain: \( x \geq 0 \)
   - Range: \( y \geq 3 \)

5. \( y = -\sqrt{x} + 3 \)
   - Domain: \( x \geq 0 \)
   - Range: \( y \leq 3 \)

6. \( y = -\sqrt{x} - 3 \)
   - Domain: \( x \geq 0 \)
   - Range: \( y \leq -3 \)

Remember What You Learned

1. Match each square root function from the list on the left with its domain and range from the list on the right.

<table>
<thead>
<tr>
<th>Function</th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = \sqrt{x} )</td>
<td>( x \geq 0 )</td>
<td>( y \geq 0 )</td>
</tr>
<tr>
<td>( y = -\sqrt{x} )</td>
<td>( x \geq 0 )</td>
<td>( y \leq 0 )</td>
</tr>
<tr>
<td>( y = \sqrt{x} - 3 )</td>
<td>( x \geq 0 )</td>
<td>( y \geq -3 )</td>
</tr>
<tr>
<td>( y = \sqrt{x} + 3 )</td>
<td>( x \geq 0 )</td>
<td>( y \geq 3 )</td>
</tr>
<tr>
<td>( y = -\sqrt{x} + 3 )</td>
<td>( x \geq 0 )</td>
<td>( y \leq 3 )</td>
</tr>
<tr>
<td>( y = -\sqrt{x} - 3 )</td>
<td>( x \geq 0 )</td>
<td>( y \leq -3 )</td>
</tr>
</tbody>
</table>

2. Graph the graph of the inequality \( y = \sqrt{x} + 6 \) as a shaded region. Which of the following points lie inside this region? \((1, 6), (2, 6), (3, 6), (1, 7), (2, 7), (3, 7)\)
   - Inside the shaded region: \((1, 6), (2, 6), (3, 6)\)
   - Outside the shaded region: \((1, 7), (2, 7), (3, 7)\)

Remember to form a square root function, choose either the positive or negative square root. For example, \( y = \sqrt{x} \) and \( y = -\sqrt{x} \) are two separate functions.
Answers (Lesson 7-3)

Skills Practice

Square Root Functions and Inequalities

Graph each function. State the domain and range of each function.

1. \( y = \sqrt{x} \)
2. \( y = -\sqrt{x} \)
3. \( y = 2\sqrt{x} \)
4. \( y = \sqrt{x} + 3 \)
5. \( y = \sqrt{x} - 3 \)
6. \( y = \sqrt{x} + 1 \)
7. \( y \leq \sqrt{x} \)
8. \( y \geq \sqrt{x} \)

Square Root Inequalities

The square root of a variable expression is an inequality that contains a square root of a variable expression. Use what you know about graphing square root functions and quadratic inequalities to graph square root inequalities.

Example

Graph the related equation \( y = \sqrt{x} \). Since the boundary should be included, the graph should be solid.

The domain includes values for \( x \) of \( x \geq 0 \), so the graph is to the right.

Exercise

1. \( y < \sqrt{x} \)
2. \( y > \sqrt{x} \)
3. \( y < 3\sqrt{x} \)
4. \( y > 3\sqrt{x} \)
5. \( y < 2\sqrt{x} - 1 \)
6. \( y > 2\sqrt{x} - 1 \)
7. \( y < \sqrt{x} - 4 \)
8. \( y > \sqrt{x} - 4 \)
9. \( y \leq \sqrt{x} + 1 \)
10. \( y \geq \sqrt{x} + 1 \)
Rachel and Ashley are testing one another’s reflexes. Rachel drops a ruler from a given height so that it falls between Ashley’s thumb and index finger. Ashley tries to catch the ruler before it falls through her hand. The time required to catch the ruler is given by \( t = \frac{\sqrt{d}}{2g} \), where \( d \) is the distance measured in feet. Complete the table. Round your answers to the nearest hundredth.

<table>
<thead>
<tr>
<th>Distance (in.)</th>
<th>Reflex Time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 in.</td>
<td>0.18</td>
</tr>
<tr>
<td>9 in.</td>
<td>0.22</td>
</tr>
<tr>
<td>12 in.</td>
<td>0.25</td>
</tr>
</tbody>
</table>

11. WEIGHT Use the formula \( d = \sqrt{\frac{3960 \cdot W}{W_f}} \), which relates distance from Earth \( d \) in miles to weight. If an astronaut’s weight on Earth \( W_e \) is 148 pounds and in space \( W_f \) is 115 pounds, how far from Earth is the astronaut? \( \text{about 532 mi} \)
7-4 Lesson Reading Guide

nth Roots

Get Ready for the Lesson

Read the introduction to Lesson 7-4 in your textbook.

Abasketball has a volume of about 382 cubic inches. Explain how you would find the radius of the basketball using a calculator. (Do not actually calculate the radius.)

Sample answer: Using a calculator, find the product of 3 times the volume. Divide this number by 4 π. Then find the positive cube root result. Round the answer to the nearest tenth.

Read the Lesson

1. For each radical below, identify the radicand and the index.
   a. \(\sqrt[3]{23}\) radicand: \(23\) index: \(3\)
   b. \(\sqrt[5]{125x^2}\) radicand: \(125x^2\) index: \(5\)
   c. \(\sqrt[4]{-343}\) radicand: \(-343\) index: \(4\)

2. Complete the following table. (Do not actually find any of the indicated roots.)

<table>
<thead>
<tr>
<th>Number</th>
<th>Number of Positive Square Roots</th>
<th>Number of Negative Square Roots</th>
<th>Number of Positive Cube Roots</th>
<th>Number of Negative Cube Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>−16</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

3. State whether each of the following is true or false.
   a. A negative number has no real fourth roots. true
   b. \(±\sqrt{121}\) represents both square roots of 121. true
   c. When you take the fifth root of \(x^3\), you must take the absolute value of \(x\) to identify the principal fifth root. false

Remember What You Learned

4. What is an easy way to remember that a negative number has no real square roots but has one real cube root? Sample answer: The square of a positive or negative number is positive, so there is no real number whose square is negative. However, the cube of a negative number is negative, so a negative number has one real cube root, which is a negative number.
7-4 Study Guide and Intervention
nth Roots

Simplify Radicals

Square Root
For any real numbers a and b, if \( a^2 = b \), then \( a \) is a square root of \( b \).

nth Root
For any real numbers a and b, and any positive integer \( n \), if \( a^n = b \), then \( a \) is an \( n \)th root of \( b \).

Real nth Roots of \( b \), \( \sqrt[n]{b} \)

1. If \( n \) is even and \( b > 0 \), then \( \sqrt[n]{b} \) has one positive root and one negative root.
2. If \( n \) is odd and \( b > 0 \), then \( \sqrt[n]{b} \) has one positive root.
3. If \( n \) is even and \( b < 0 \), then \( \sqrt[n]{b} \) has no real roots.
4. If \( n \) is odd and \( b < 0 \), then \( \sqrt[n]{b} \) has one real root.

Example 1 Simplify \( \sqrt[4]{64} \).

\( \sqrt[4]{64} = \sqrt[4]{(2)^4} = 2 \)

Example 2 Simplify \( \sqrt[3]{(2a - 1)^3} \).

\( \sqrt[3]{(2a - 1)^3} = 2a - 1 \)

Exercises

Simplify.

1. \( \sqrt{81} \)
2. \( \sqrt[3]{-27} \)
3. \( \sqrt[4]{144} \)
4. \( \sqrt[3]{8} \)
5. \( \sqrt[4]{64} \)
6. \( \sqrt[3]{1} \)
7. \( \sqrt[4]{16} \)
8. \( \sqrt[3]{64} \)
9. \( \sqrt[4]{1} \)
10. \( \sqrt[3]{27} \)
11. \( \sqrt[4]{81} \)
12. \( \sqrt[3]{9} \)
13. \( \sqrt[4]{81} \)
14. \( \sqrt[3]{64} \)
15. \( \sqrt[4]{1} \)
16. \( \sqrt[3]{1} \)
17. \( -\sqrt{0.04} \)
18. \( \sqrt[4]{(-4)^4} \)
19. \( \sqrt[4]{16} \)
20. \( \sqrt[3]{25} \)
21. \( \sqrt[4]{25} \)
22. \( \sqrt[3]{(x - 1)^3} \)
23. \( \sqrt[4]{(m - 5)^4} \)

Answers

1. 9
2. -7
3. 12
4. \( \sqrt[4]{16} = 2 \)
5. \( \sqrt[3]{8} = 2 \)
6. \( \sqrt[4]{1} = 1 \)
7. \( \sqrt[3]{1} = 1 \)
8. \( \sqrt[4]{81} = 3 \)
9. \( -1 \)
10. \( \sqrt[3]{27} = 3 \)
11. \( \sqrt[4]{81} = 3 \)
12. \( \sqrt[3]{9} = 2 \)
13. \( \sqrt[3]{64} = 4 \)
14. \( \sqrt[4]{16} = 2 \)
15. \( \sqrt[4]{1} = 1 \)
16. \( -0.3 \)
17. not a real number
18. \( \sqrt[4]{(-4)^4} = 4 \)
19. \( \sqrt[4]{25} = 5 \)
20. \( \sqrt[3]{25} = 5 \)

Use a calculator to approximate each value to three decimal places.

1. \( \sqrt{62} \)
2. \( \sqrt{1050} \)
3. \( \sqrt{0.054} \)
4. \( \sqrt{5.45} \)
5. \( \sqrt{3280} \)
6. \( \sqrt{18,600} \)
7. \( \sqrt{0.095} \)
8. \( \sqrt{15} \)
9. \( \sqrt{100} \)
10. \( \sqrt{5} \)
11. \( \sqrt{3200} \)
12. \( \sqrt{0.06} \)
13. \( \sqrt{12,500} \)
14. \( \sqrt{0.46} \)
15. \( \sqrt{500} \)
16. \( \sqrt{0.15} \)
17. \( \sqrt{4200} \)
18. \( \sqrt{55} \)

19. LAW ENFORCEMENT The formula \( r = 2\sqrt{L} \) is used by police to estimate the speed \( r \) in miles per hour of a car if the length \( L \) of the car’s skid mark is measured in feet. Estimate to the nearest tenth of a mile per hour the speed of a car that leaves a skid mark 300 feet long.

20. SPACE TRAVEL The distance to the horizon \( d \) miles from a satellite orbiting \( h \) miles above Earth can be approximated by \( d = \sqrt{8000h + h^2} \). What is the distance to the horizon if a satellite is orbiting 150 miles above Earth? About 1100 mi
Lesson 7-4
Skills Practice

nth Roots

Use a calculator to approximate each value to three decimal places.

1. √220 ≈ 15.166
2. √38 ≈ 6.164
3. -√162 ≈ -12.292
4. √5.6 ≈ 2.366
5. √88 ≈ 9.380
6. √-222 ≈ -14.907
7. -√0.34 ≈ -0.583
8. √900 ≈ 3.000

Simplify.
9. ±√51 = ±7.143
10. √144 = 12
12. √-57 is not a real number
13. √0.36 = 0.6
14. -√1/9 = -2/3
15. √-8 = -2
16. -√27 = -3
17. √0.064 = 0.4
18. √32 = 2
19. √61 = 3
20. √y

21. √125 = 5
22. √64r2 = 8r
23. √-27d2 = -3a
24. √m + 1 = m + 1
25. √-100p2q2 = -10pq
26. √16w + 5 = 2w
27. √(-3k) = 9c
28. √(a + b) = |a + b|

7-4 Practice
nth Roots

Use a calculator to approximate each value to three decimal places.

1. √7.8 ≈ 2.793
2. -√89 ≈ -9.434
3. √25 ≈ 2.924
4. -√24 ≈ -1.587
5. √1.1 ≈ 1.024
6. √-0.1 = -0.314
7. √555 ≈ 24.080
8. √0.947 ≈ 0.970

Simplify.
9. √0.81
10. -√324
11. -√256
12. √64
13. √-64
14. √0.512
15. √-243
16. √-1296
17. √(1024/343) ≈ 3.266
18. √243x10 ≈ 14.14
19. (14x2) = 36x
20. (14x2) / 14 = not a real number
21. √49m7n4 = 7mnt
22. √16x25 = 40x
23. √-64y6z8 = -4r
24. √(2x)3
25. √-62x5 = 6x
26. √-216y3z = 6y2z
27. √-766x4y = -4xy
28. √-27x5 = 3x
29. √-14mn2 = -2x
30. √-32x5z = -2x
31. √(m + 4) = m + 4
32. √(2 + 1) = 2x
33. √-49v16x4 = 7d
34. √(x - 5) = 7d
35. √(a + b) = 7d
36. √(a + b) = 7d

37. RADIANT TEMPERATURE Thermal sensors measure an object's radiant temperature, which is the amount of energy radiated by the object. The internal temperature of an object is called its kinetic temperature. The formula T = T + T relates an object's radiant temperature T to its kinetic temperature T. The variable e is the formula is a measure of how well the object radiates energy. If an object's kinetic temperature is 30°C and e = 0.94, what is the object's radiant temperature to the nearest tenth of a degree?

29.5°C

38. HERO’S FORMULA Salvatore is buying fertilizer for his triangular garden. He knows the lengths of all three sides, so he is using Hero’s formula to find the area. Hero’s formula states that the area of a triangle is √(s(s - a)(s - b)(s - c)), where a, b, and c are the lengths of the sides of the triangle and s is half the perimeter of the triangle. If the lengths of the sides of Salvatore’s garden are 15 feet, 17 feet, and 20 feet, what is the area of the garden? Round your answer to the nearest whole number. 124 ft²
1. CUBES Cathy is building a cubic storage room. She wants the volume of the space to be 1728 cubic feet. What should the dimensions of the cube be?

12 ft by 12 ft by 12 ft

2. ASTRONOMY A special form of Kepler's Third Law of Planetary Motion is given by \( a = \sqrt{P^3} \) where \( a \) is the average distance of an object from the Sun in AU (astronomical units) and \( P \) is the period of the orbit in years. If an object is orbiting the Sun with a period of 12 years, what is its distance from the Sun?

524 AU

3. TUNING Two notes are an octave apart if the frequency of the higher note is twice the frequency of the lower note. Casey is experimenting with an instrument that has 6 notes tuned so that the frequency of each successive note increases by the same factor and the first and last note are an octave apart. By what factor does the frequency increase from note to note?

\( \sqrt[2]{2} \) or approximately 1.15

4. MARKUPS A wholesaler manufactures a part for \( D \) dollars. The wholesaler sells the part to a dealer for a \( P \) percent markup. The dealer sells the part to a retailer at an additional \( P \) percent markup. The retailer in turn sells the part to its customers marking up the price yet another \( P \) percent. What is the price that customers see? If the customer buys the part for \( $80 \) and the markup is 40%, what approximately was the original cost to make the part?

\( D(1 + P)^3; $29.15 \)

5. One group in the class made a 2-foot long pendulum. Use the formula to determine how long it will take for their pendulum to swing back and forth.

1.57 seconds

6. Another group decided they wanted to make a pendulum that took about 1.76 seconds to go back and forth. Approximately how long should their pendulum be?

2.5 feet

7. Kepler's Third Law of Planetary Motion

Example

Use the formula \( \sqrt{a^2 \pm b} \approx a \pm \frac{b}{2a} \) to approximate \( \sqrt{101} \) and \( \sqrt{622} \).

a. \( \sqrt{101} \approx 10 + \frac{1}{10} \approx 10.05 \)

b. \( \sqrt{622} \approx 25 + \frac{3}{250} \approx 25.012 

Exercise

Use the formula to find an approximation for each square root to the nearest hundredth. Check your work with a calculator.

1. \( \sqrt{236} \)

2. \( \sqrt{95} \)

3. \( \sqrt{402} \)

4. \( \sqrt{1604} \)

5. \( \sqrt{223} \)

6. \( \sqrt{80} \)

7. \( \sqrt{4890} \)

8. \( \sqrt{2505} \)

9. \( \sqrt{3575} \)

10. \( \sqrt{144100} \)

11. \( \sqrt{290} \)

12. \( \sqrt{86} \)

13. Show that \( a - \frac{b}{2a} \approx \sqrt{a^2 - b} \) for \( a > b \). Disregard \( \frac{b^2}{4a^2} \) for \( a > b \).

\( a - \frac{b}{2a} \approx \sqrt{a^2 - b} \approx \sqrt{a^2 - b} \)
7-5 Lesson Reading Guide

Operations with Radical Expressions

Get Ready for the Lesson

Read the introduction to Lesson 7-5 in your textbook.

Describe how you could use the golden ratio to find the height of a golden triangle if you knew its width. Sample answer: Use a calculator to multiply the width by 2 and divide the result by the quantity of \( \sqrt{5} - 1 \). Round this answer to the nearest tenth.

Read the Lesson

1. Complete the conditions that must be met for a radical expression to be in simplified form.
   - The \( n \) is as small as possible.
   - The radicand contains no factors (other than 1) that are \( n \)th powers of an integer or polynomial.
   - The radicand contains no fractions.
   - No radicals appear in the denominator.

2. a. What are conjugates of radical expressions used for? To rationalize binomial denominators

   b. How would you use a conjugate to simplify the radical expression \( \frac{1 + \sqrt{2}}{3 - \sqrt{2}} \)?

   Multiply numerator and denominator by \( 3 + \sqrt{2} \).

   c. In order to simplify the radical expression in part b, two multiplications are necessary. The multiplication in the numerator would be done by the FOIL method, and the multiplication in the denominator would be done by finding the difference of two squares.

Remember What You Learned

3. One way to remember something is to explain it to another person. When rationalizing the denominator in the expression \( \frac{1}{\sqrt{2}} \), many students think they should multiply numerator and denominator by \( \frac{\sqrt{2}}{\sqrt{2}} \). How would you explain to a classmate why this is incorrect and what he should do instead? Sample answer: Because you are working with cube roots, not square roots, you need to make the radicand in the denominator a perfect cube, not a perfect square. Multiply numerator and denominator by \( \frac{\sqrt{4}}{\sqrt{4}} \) to make the denominator \( \sqrt{8} \), which equals 2.

7-5 Study Guide and Intervention

Operations with Radical Expressions

Simplify Radical Expressions

Remember What You Learned

Exercises

1. \( 5\sqrt{54} \), \( 15\sqrt{6} \)
2. \( \sqrt{32a^{2}b^{5}} \), \( 2a^{2}b^{5}\sqrt{2a} \)
3. \( \sqrt{75xy^3} \), \( 5x^{2}y^{3}\sqrt{3y} \)
4. \( \frac{36}{25} \), \( \sqrt{6}\sqrt{5} \)
5. \( \sqrt{\frac{a^{2}b^{3}}{36}} \), \( \frac{a^{2}b^{3}}{36} \sqrt{3} \)
6. \( \sqrt{\frac{p^{4}q^{2}}{40}} \), \( pq\sqrt{25p^{2}} \)
Answers (Lesson 7-5)

Operations with Radical Expressions

Example 1: Simplify.

1. \( \sqrt{2} \cdot \sqrt{8} \)
2. \( \sqrt{5} \cdot \sqrt{3} \)
3. \( \sqrt{2} \cdot \sqrt{2} \)
4. \( \sqrt{a} \cdot \sqrt{b} \)
5. \( \sqrt{a^2} \)
6. \( \sqrt{a^2b^2} \)

Example 2: Multiply.

1. \( \sqrt{2} \cdot \sqrt{2} \)
2. \( \sqrt{3} \cdot \sqrt{3} \)
3. \( \sqrt{5} \cdot \sqrt{5} \)
4. \( \sqrt{7} \cdot \sqrt{7} \)

Example 3: Simplify.

1. \( \sqrt{25} \cdot \sqrt{9} \)
2. \( \sqrt{16} \cdot \sqrt{4} \)
3. \( \sqrt{81} \cdot \sqrt{16} \)
4. \( \sqrt{100} \cdot \sqrt{100} \)

Exercises

17. \( 5 \sqrt{2} \cdot 2 \sqrt{3} \)
18. \( 3 \sqrt{2} \cdot 2 \sqrt{3} \)
19. \( 3 \sqrt{2} \cdot 4 \sqrt{3} \)
20. \( 5 \sqrt{2} \cdot 3 \sqrt{3} \)

Chapter 7

Glencoe Algebra 2
1. Cubes Cathy has a rectangular box with dimensions 20 inches by 35 inches by 40 inches. She would like to replace it with a box in the shape of a cube but with the same volume. What should the length of a side of the cube be? Express your answer as a radical expression in simplest form.

10√28 in.

2. Physics The speed of a wave traveling over a string is given by where $v$ is the tension of the string and $u$ is the density. Rewrite the expression in simplest form by rationalizing the denominator. $\sqrt{\frac{v}{u}}$

3. Tuning With each note higher on a piano, the frequency of the pitch increases by a factor of $\sqrt{2}$. What is the ratio of the frequencies of two notes that are 6 steps apart on the piano? What is the ratio of the frequencies of two notes that are 9 steps apart on the piano? Express your answers in simplest form. $\sqrt{2}$ and $\sqrt{3}$

4. Lights Suppose a light has a brightness intensity of $I_1$ when it is at a distance of $d_1$ and a brightness intensity of $I_2$ when it is at a distance of $d_2$. These quantities are related by the equation $\frac{d_1^2}{d_2^2} = \frac{I_2}{I_1}$. Suppose $I_1 = 50$ units and $I_2 = 24$ units. What would $\frac{d_1}{d_2}$ be? Express your answer in simplest form. $\frac{5}{6}$

5. Braking The formula $s = 2\sqrt{vt}$ estimates the speed $s$ in miles per hour of a car when it leaves skid marks $t$ feet long. Use the formula to write a simplified expression for the measure of the hypotenuse $3x^2 + y\sqrt{13}$.
Chapter 7

Rational Exponents

Read the introduction to Lesson 7-6 in your textbook. The formula in the introduction contains the exponent \( \frac{2}{5} \). What do you think it might mean to raise a number to the \( \frac{2}{5} \) power?

Sample answer: Take the fifth root of the number and square it.

Read the Lesson

1. Complete the following definitions of rational exponents.
   - For any real number \( b \) and for any positive integer \( n \), \( b^{\frac{1}{n}} = \sqrt[n]{b} \) except when \( b < 0 \) and \( n \) is even.
   - For any nonzero real number \( b \), and any integers \( m \) and \( n \), with \( n > 1 \), \( b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left( \sqrt[n]{b} \right)^m \), except when \( b < 0 \) and \( n \) is even.

2. Complete the conditions that must be met in order for an expression with rational exponents to be simplified.
   - It has no negative exponents.
   - It has no fractional exponents in the denominator.
   - It is not a complex fraction.
   - The index of any remaining radical is the least number possible.

3. Margarita and Pierre were working together on their algebra homework. One exercise asked them to evaluate the expression \( 27^{\frac{1}{3}} \). Margarita thought that they should raise 27 to the fourth power first and then take the cube root of the result. Pierre thought that they should take the cube root of 27 first and then raise the result to the fourth power. Whose method is correct? Both methods are correct.

Remember What You Learned

4. Some students have trouble remembering which part of the fraction in a rational exponent gives the power and which part gives the root. How can your knowledge of integer exponents help you to keep this straight?

Sample answer: An integer exponent can be written as a rational exponent. For example, \( 2^3 = 2^{\frac{3}{1}} \). You know that this means that 2 is raised to the third power, so the numerator must give the power, and, therefore, the denominator must give the root.
Rational Exponents and Radicals

Definition of \( b^{\frac{1}{n}} \) For any real number \( b \) and any positive integer \( n \),
\[ b^{\frac{1}{n}} = \sqrt[n]{b} \], except when \( b < 0 \) and \( n \) is even.

Definition of \( b^{\frac{m}{n}} \) For any nonzero real number \( b \) and any integers \( m \) and \( n \), with \( n > 1 \),
\[ b^{\frac{m}{n}} = \left( \sqrt[n]{b} \right)^m \], except when \( b < 0 \) and \( n \) is even.

Example 1
Write 28\(^{\frac{1}{3}}\) in radical form.

Notice that 28 > 0.
\[ 28^{\frac{1}{3}} = \sqrt[3]{28} \]

Example 2
Evaluate \( -8^{\frac{1}{3}} \).

Notice that \(-8 < 0, \) \(-125 < 0, \) and \( 3 \) is odd.
\[ (-8)^{\frac{1}{3}} = \sqrt[3]{-8} = -2 \]

Exercises

- \( \sqrt{11} \)
- \( \sqrt[5]{15} \)
- \( \sqrt[10]{3000} \)

Write each expression in radical form.
1. \( 11^{\frac{1}{2}} \)
2. \( 15^{\frac{1}{5}} \)
3. \( 3000^{\frac{1}{10}} \)

Write each radical using rational exponents.
4. \( \sqrt{47} \)
5. \( \sqrt[3]{3x^5} \)
6. \( \sqrt[6]{162p^5} \)
7. \( \sqrt{x^2} \)
8. \( \sqrt{y^3} \)
9. \( \sqrt{p^5} \)
10. \( \sqrt[4]{128} \)
11. \( \sqrt[5]{49} \)
12. \( \sqrt[9]{288} \)
13. \( \sqrt[2]{32} \cdot 3 \sqrt[16]{16} \)
14. \( \sqrt[25]{25} \cdot \sqrt[125]{125} \)
15. \( \sqrt[16]{16} \)
16. \( \frac{x^{\frac{3}{2}}}{\sqrt{y^2}} \)
17. \( \sqrt[6]{48} \)
18. \( \sqrt[3]{x^2 \cdot y^2} \)

Evaluate each expression.
7. \( -27^{\frac{1}{3}} \)
8. \( \frac{5^{\frac{1}{5}}}{2^{\frac{1}{2}}} \)
9. \( (0.0004)^{\frac{1}{3}} \)

Simplify each expression.
1. \( x^6 \cdot x^{\frac{1}{2}} \)
2. \( \left( y^3 \right)^{\frac{1}{3}} \)
3. \( p^{\frac{1}{2}} \cdot p^{\frac{1}{3}} \)
4. \( \left( \frac{m^{\frac{1}{2}}}{n^{\frac{3}{2}}} \right)^{\frac{1}{3}} \)
5. \( x^{\frac{3}{4}} \cdot x^{\frac{1}{3}} \)
6. \( \left( \frac{a^{2}}{b^{3}} \right)^{\frac{1}{2}} \)
7. \( \frac{p^{\frac{1}{2}}}{p^{\frac{1}{2}}} \)
8. \( \frac{x^{\frac{3}{4}}}{y^{\frac{3}{4}}} \)
9. \( \frac{3^{\frac{1}{2}}}{2^{\frac{1}{2}}} \)
10. \( \frac{2^{\frac{1}{2}}}{2^{\frac{1}{2}}} \)
11. \( \sqrt{7} \)
12. \( \sqrt{9} \)
13. \( \sqrt{32} \cdot 3 \sqrt{16} \)
14. \( \sqrt{25} \cdot \sqrt{125} \)
15. \( \sqrt{16} \)
16. \( \frac{x^{\frac{3}{2}}}{\sqrt[3]{y^2}} \)
17. \( \sqrt[6]{48} \)
18. \( \sqrt[3]{x^2 \cdot y^2} \)
7-6 Skills Practice

Rational Exponents

Write each expression in radical form.
1. \(3^{\frac{1}{3}} \sqrt[3]{3}\)
2. \(8^{\frac{1}{3}} \sqrt[3]{8}\)
3. \(12^{\frac{3}{2}} \text{ or } (\sqrt{12})^3 \text{ or } 2\sqrt{18}\)
4. \((a^3)^{\frac{1}{3}} a\sqrt[3]{a}\)

Write each radical using rational exponents.
5. \(\sqrt{5} 5^{\frac{1}{2}}\)
6. \(\sqrt{37} 37^{\frac{1}{2}}\)
7. \(\sqrt{15} 15^{\frac{1}{2}}\)
8. \(\sqrt[6]{x^3} 6x^\frac{3}{2}\)

Evaluate each expression.
9. \(32^{\frac{1}{5}} 2\)
10. \(81^{\frac{1}{3}} 3\)
11. \(27^{\frac{1}{3}} \frac{3}{3}\)
12. \(4^{\frac{3}{2}} \frac{4}{2}\)
13. \(16^{\frac{1}{2}} 64\)
14. \((-243)^{\frac{1}{5}} 81\)
15. \(27^{\frac{1}{3}} \cdot 729\)
16. \(\left(\frac{1}{9}\right)^{\frac{1}{2}} \frac{8}{27}\)

Simplify each expression.
17. \(c^{\frac{2}{3}} c^1 c^3\)
18. \(m^{\frac{3}{2}} m^5 m^2\)
19. \(\left(\frac{q}{p}\right)^{\frac{3}{4}} q^\frac{3}{4}\)
20. \(p^{\frac{4}{3}} \frac{1}{p^4} \text{ or } \frac{p^4}{p^3}\)
21. \(x^{\frac{3}{4}} \frac{1}{x^2} \text{ or } \frac{x^{\frac{3}{4}}}{x^2}\)
22. \(x^{\cdot \frac{3}{2}} \frac{1}{x^{\frac{1}{2}}} \text{ or } \frac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}}\)
23. \(x^{\frac{1}{2}} \frac{1}{y^{\frac{1}{2}}} \text{ or } \frac{x^{\frac{1}{2}}}{y^{\frac{1}{2}}}\)
24. \(\frac{1}{n^2} \frac{1}{m^2} \text{ or } \frac{1}{n^2 m^2}\)
25. \(\sqrt[6]{a^3 b^3} \sqrt[6]{a^3 b^3}\)
26. \(\sqrt[6]{a^3 b^3} \sqrt[6]{a^3 b^3}\)

Evaluate each expression.
9. \(81^{\frac{1}{3}} 3\)
10. \(1024^{\frac{1}{4}} \frac{1}{4}\)
11. \(8^{\frac{1}{3}} \frac{1}{32}\)
12. \(-256^{\frac{1}{3}} \frac{1}{64}\)
13. \((-64)^{\frac{1}{4}} \frac{1}{16}\)
14. \(27^{\frac{1}{3}} \frac{27}{243}\)
15. \((\frac{25}{36})^{\frac{1}{4}} \frac{25}{36}\)
16. \(\frac{64}{343} \frac{16}{49}\)
17. \(\left(\frac{25}{64} \cdot 64\right)^{-\frac{1}{4}} \frac{-5}{4}\)

Simplify each expression.
18. \(a^2 \cdot g^2 \frac{g}{g}\)
19. \(s^2 \cdot s^{-1} \frac{s}{s}\)
20. \((a^{-2})^{-\frac{1}{4}} \frac{a}{u}\)
21. \(y^{\frac{1}{2}} \frac{1}{y^{\frac{1}{2}}} \text{ or } \frac{y^{\frac{1}{2}}}{y^{\frac{1}{2}}}\)
22. \(b^{\frac{4}{3}} \frac{1}{b^4} \text{ or } \frac{b^4}{b^3}\)
23. \(q^{\frac{2}{3}} \frac{1}{q^2} \text{ or } \frac{q^2}{q^2}\)
24. \(\frac{1}{x^2} \frac{1}{y^2} \frac{1}{z} \frac{1}{z-1}\)
25. \(\frac{2x}{x^2} \frac{2z}{z-1}\)
26. \(\sqrt[6]{5} \frac{2\sqrt{2}}{2}\)
27. \(\sqrt[6]{12} \cdot \sqrt[6]{12}\)
28. \(\sqrt[6]{6} \cdot \frac{3\sqrt[6]{6}}{12\sqrt[6]{12}}\)
29. \(\sqrt[6]{6} \cdot \frac{3\sqrt[6]{6}}{3\sqrt[6]{6}}\)

30. ELECTRICITY The amount of current in amperes \(I\) that an appliance uses can be calculated using the formula \(I = \frac{P}{R}\), where \(P\) is the power in watts and \(R\) is the resistance in ohms. How much current does an appliance use if \(P = 500\) watts and \(R = 10\) ohms? Round your answer to the nearest tenth. \(7.1\) amps

31. BUSINESS A company that produces DVDs uses the formula \(C = 88n^{\frac{1}{3}} + 330\) to calculate the cost \(C\) in dollars of producing \(n\) DVDs per day. What is the company’s cost to produce 150 DVDs per day? Round your answer to the nearest dollar. \$798
How many cells were in the culture after 20 minutes? Express your answer in simplest form.

$$200 \cdot 15^3$$

After 20 minutes, express your answer in simplest form.

$$20^3 \cdot 15$$
When an asset such as a house increases in value over time, it is said to **appreciate**. If the value increases by a fixed percent each year, or other period of time, the amount **\( y \)** that quantity after **\( t \) years** is given by

\[
y = a(1 + r)^t,
\]
where **\( a \)** is the initial amount and **\( r \)** is the percent of increase expressed as a decimal. You can use a spreadsheet to investigate future values of an asset.

**Example**

Michael Blackstock is considering buying a piece of investment property for $85,000. The homes in the area are appreciating at an average rate of 4% per year. Find the expected value of the home in 1 year, 1 year and 6 months, 4 years, and 6 years and 9 months.

Use rows 1 and 2 to enter the initial amount and the rate of increase. Then use Column A to enter the amounts of time. Enter the numbers of months as a fraction of a year since **\( t \)** is measured in years. Column B contains the formulas for the value of the home.

Format the cells containing the values as currency so that they are displayed as dollars and cents. The expected value of the home after each amount of time is shown in the spreadsheet.

**Exercises**

1. If Mr. Blackstock chooses another property in the neighborhood that costs $99,900, what are the expected values of that home in the same periods of time? $103,896.00, $105,953.55, $116,868.87, $130,178.88
2. What would Mr. Blackstock’s profit be on the $99,900 home if he sold it after 9 years and 3 months? $43,689.89
3. If an antique chair worth $165.00 increases in value an average of 3 1/2% every year, how much will it be worth next year? $170.78
4. Often assets like cars decrease in value over time. This asset is said to **depreciate**. If the value decreases by a fixed percent each year, or other period of time, the amount **\( y \)** that quantity after **\( t \) years** is given by

\[
y = a(1 - r)^t,
\]
where **\( a \)** is the initial amount and **\( r \)** is the percent of decrease expressed as a decimal. Use a spreadsheet to find the value of a car purchased for $18,500 after 2 years, 2 years and 6 months, and 4 years and 3 months if the car depreciates at a rate of 12% per year.
Example 1
Solve $2\sqrt{4x} + 8 - 4 = 8$.

Step 1: Isolate the radical on one side of the equation.

Step 2: Divide each side by 4.

Step 3: Solve the radical inequality.

Example 2
Solve $\sqrt{3x} + 1 = \sqrt{5x} - 1$.

Step 1: Isolate the radical on one side of the equation.

Step 2: Divide each side by 2.

Step 3: Solve the radical inequality.

To eliminate the radical, raise each side of the equation to a power equal to the index of the radical.

Check values to check your solution.

Example 3
Solve $\sqrt{3(0) + 1} - 1 = 0$.

Since the radicand of a square root must be greater than or equal to zero, first solve $\sqrt{3(0) + 1} - 1 = 0$. Then solve $\sqrt{3(0) + 1} - 1 = 1$. It appears that $-\frac{1}{2} \leq x \leq 3$ is the solution. Test some values.

<table>
<thead>
<tr>
<th>$x = -1$</th>
<th>$x = 0$</th>
<th>$x = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{3(-1) + 1} - 1 = 0$ is not a solution.</td>
<td>$\sqrt{3(0) + 1} - 1 = 0$ is a solution.</td>
<td>$\sqrt{3(4) + 1} - 1 = 0$ is not a solution.</td>
</tr>
</tbody>
</table>

Therefore, the solution $-\frac{1}{2} \leq x \leq 3$ checks.

Exercises
Solve each equation.

1. $\sqrt{3} + 2\sqrt{3} - 5 = \frac{\sqrt{3}}{3}$
   - $x = 15$

2. $2\sqrt{3}x + 4 + 1 = 15$
   - $x = 7$

3. $8 + \sqrt{x + 1} - 2 = 8$
   - No solution

4. $\sqrt{5} - x - 4 - 6 = -9$
   - No solution

5. $2\sqrt{2}x + 1 = 8$
   - No solution

6. $\sqrt{2}x + 1 = 8$
   - $x = 4$

7. $\sqrt{2}x + 1 = 8$
   - $x = 3$

8. $\sqrt{2}x + 1 = 8$
   - $x = 2$

9. $\sqrt{2}x + 1 = 8$
   - $x = 1$

10. $\sqrt{2}x + 1 = 8$
    - $x = 0$

11. $\sqrt{2}x + 1 = 8$
    - $x = -1$

12. $\sqrt{2}x + 1 = 8$
    - $x = 2$

Solve each inequality.

1. $\sqrt{2}x + 1 = 8$
   - $\sqrt{2}x + 1 = 8$
   - $x = 4$

2. $\sqrt{2}x + 1 = 8$
   - $x = 3$

3. $\sqrt{2}x + 1 = 8$
   - $x = 2$

4. $\sqrt{2}x + 1 = 8$
   - $x = 1$

5. $\sqrt{2}x + 1 = 8$
   - $x = 0$

6. $\sqrt{2}x + 1 = 8$
   - $x = -1$

7. $\sqrt{2}x + 1 = 8$
   - $x = 2$

8. $\sqrt{2}x + 1 = 8$
   - $x = 3$

9. $\sqrt{2}x + 1 = 8$
   - $x = 4$

10. $\sqrt{2}x + 1 = 8$
    - $x = 5$

11. $\sqrt{2}x + 1 = 8$
    - $x = 6$

12. $\sqrt{2}x + 1 = 8$
    - $x = 7$

13. $\sqrt{2}x + 1 = 8$
    - $x = 8$

14. $\sqrt{2}x + 1 = 8$
    - $x = 9$

15. $\sqrt{2}x + 1 = 8$
    - $x = 10$

16. $\sqrt{2}x + 1 = 8$
    - $x = 11$

17. $\sqrt{2}x + 1 = 8$
    - $x = 12$

18. $\sqrt{2}x + 1 = 8$
    - $x = 13$

19. $\sqrt{2}x + 1 = 8$
    - $x = 14$

20. $\sqrt{2}x + 1 = 8$
    - $x = 15$

21. $\sqrt{2}x + 1 = 8$
    - $x = 16$

22. $\sqrt{2}x + 1 = 8$
    - $x = 17$

23. $\sqrt{2}x + 1 = 8$
    - $x = 18$

24. $\sqrt{2}x + 1 = 8$
    - $x = 19$

25. $\sqrt{2}x + 1 = 8$
    - $x = 20$

26. $\sqrt{2}x + 1 = 8$
    - $x = 21$
Chapter 7

7-7 Skills Practice

Solving Radical Equations and Inequalities

Solve each equation or inequality.

1. \( \sqrt{x} = 5 \) \( \quad 25 \)
2. \( \sqrt{x} + 3 = 7 \) \( \quad 16 \)
3. \( 5\sqrt{y} = 1 \) \( \quad \frac{1}{25} \)
4. \( 6\sqrt{y} + 1 = 0 \) no solution
5. \( 18 - 3y \leq 25 \) no solution
6. \( \sqrt{2w} = 4 \) \( \quad 32 \)
7. \( \sqrt{b} - 5 = 4 \) \( \quad 21 \)
8. \( \sqrt{3n} + 1 = 5 \) \( \quad 8 \)
9. \( \sqrt{3p} - 6 = 3 \) \( \quad 11 \)
10. \( 2 + \sqrt{3p} + 7 = 6 \) \( \quad 3 \)
11. \( \sqrt{k} - 4 - 1 = 5 \) \( \quad 40 \)
12. \( 2q + 3 \sqrt{q} = 2 \) \( \quad \frac{5}{2} \)
13. \( (t - 3)^{\frac{1}{2}} = 2 \) \( \quad 11 \)
14. \( 4 - (1 - 7u)^{\frac{1}{2}} = 0 \) no solution
15. \( \sqrt{2z} - 2 = \sqrt{z} - 4 \) no solution
16. \( \sqrt{g} + 1 = \sqrt{2g} - 7 \) \( \quad 8 \)
17. \( \sqrt{s} - 1 = 4\sqrt{s} + 1 \) no solution
18. \( 5 + \sqrt{s} - 3 \leq 6 \) \( \quad s \leq 4 \)
19. \( -2 + \sqrt{3x} + 3 < 7 \) \( -1 \leq x < 26 \)
20. \( -\sqrt{2a} + 4 \geq -6 \) \( -2 \leq a \leq 16 \)
21. \( 2\sqrt{y} - 3 > 10 \) \( \quad r > 7 \)
22. \( 4 - \sqrt{3x} + 1 > 3 \) \( -\frac{1}{3} \leq x < 0 \)
23. \( \sqrt{y} + 4 - 3 \geq 3 \) \( \quad y \geq 32 \)
24. \( -3\sqrt{L} + 3 \geq -15 \) \( -\frac{3}{11} \leq r \leq 2 \)

Chapter 7

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Glencoe Algebra 2

7-7 Practice

Solving Radical Equations and Inequalities

Solve each equation or inequality.

1. \( \sqrt{x} = 8 \) \( \quad 64 \)
2. \( 4 - \sqrt{x} = 3 \) \( \quad 1 \)
3. \( \sqrt{20} + 3 = 10 \) \( \quad 9 \)
4. \( 4\sqrt{3h} - 2 = 0 \) \( \quad \frac{1}{12} \)
5. \( c^2 + 6 = 9 \) \( \quad 3 \)
6. \( 18 + 7b = 12 \) no solution
7. \( \sqrt{d} + 2 = 7 \) \( \quad 341 \)
8. \( \sqrt{w} - 7 = 1 \) \( \quad 8 \)
9. \( 6 + \sqrt{q} - 4 = 9 \) \( \quad 31 \)
10. \( \sqrt{y} - 9 + 4 = 0 \) no solution
11. \( \sqrt{2m} - 6 = 16 \) \( \quad 131 \)
12. \( \sqrt{4m + 1} - 2 = -\frac{63}{4} \)
13. \( \sqrt{8m} - 5 = 1 \) \( \quad 7 \)
14. \( \sqrt{1 - 4t} = -8 \) \( \quad 6 \)
15. \( \sqrt{2v} - 5 = 3 \) \( \quad \frac{41}{2} \)
16. \( Gv - 2t^2 + 12 - 7 \) \( \quad 11 \)
17. \( \sqrt{g} + 1 \) \( \quad 6 = 4 \) \( \quad 33 \)
18. \( \sqrt{2b} + 5 = \sqrt{2b} + 1 \) \( \quad 2 \)
19. \( 3\sqrt{u} = 12 \) \( \quad a = 16 \)
20. \( \sqrt{v} - 6 \) \( \quad \sqrt{2} \)
21. \( \sqrt{6x} - 4 = \sqrt{2x + 10} \) \( \quad \frac{7}{2} \)
22. \( 4\sqrt{2} + 5 = \sqrt{2x + 1} \) \( \quad 6 \)
23. \( 3\sqrt{a} \leq 12 \) \( \quad a \leq 16 \)
24. \( \sqrt{z} + 5 = 4 \leq 13 \) \( -5 \leq z \leq 76 \)
25. \( 8 + \sqrt{2} \leq 5 \) \( \quad 3 \)
26. \( \sqrt{2s} - 3 \leq \frac{3}{2} \leq a \leq 14 \)
27. \( 9 - \sqrt{4} \leq 6 \) \( c \geq 5 \)
28. \( \sqrt{x} - 1 < -2 \) \( x < -7 \)
29. STATISTICS Statistics use the formula \( \sigma = \sqrt{\bar{x}} \) to calculate a standard deviation \( \sigma \), where \( \sigma \) is the variance of a data set. Find the variance when the standard deviation is 15. \( \quad 225 \)
30. GRAVITATION Helena drops a ball from 25 feet above a lake. The formula \( t = \frac{1}{2} \sqrt{\frac{d}{h}} \) describes the time \( t \) in seconds that the ball is \( h \) feet above the water. How many feet above the water will the ball be after 1 second? \( 9 \) ft
7-7 Word Problem Practice

Rational Equations and Inequalities

1. SIGNS A sign painter must spend $800 + 400 to make n signs. How many signs can the painter make for $1200? 1000

2. LATERAL AREA The lateral area of a cone with base radius r and height h is given by the formula \( L = \pi \sqrt{r^2 + h^2} \). A cone has a lateral area of 65\( \pi \) square units and a base radius of 5 units.

What is the height of the cone? 12 units

3. ORIGAMI Georgia wants to fold a square piece of paper into an equilateral triangle. She wants to locate the distance x up the side of the square where she can make the fold indicated by the dashed line in the figure so that \( a = b \). From geometry class, she knows that \( a = \sqrt{1 + x^2} \) and \( b = \sqrt{2} - x \).

\( 2 - \sqrt{3} \)

4. TETHERS A tether is being attached to a 25-foot pole in such a way that \( x + y = 50 \). By the Pythagorean Theorem, the distance \( y = \sqrt{x^2 + 25^2} \). What must x be?

18.75 ft

5. Write an expression that gives the distance of the asteroid from Earth as a function of x.

\( \sqrt{x^2 + \frac{289}{x^2}} \)

6. For what values of x will the asteroid be in range of Carl’s telescope?

\( 17 \leq x \leq 12 \)

7-7 Enrichment

Truth Tables

In mathematics, the basic operations are addition, subtraction, multiplication, division, finding a root, and raising to a power. In logic, the basic operations are the following: not (\( \lnot \)), and (\( \land \)), or (\( \lor \)), and implication (\( \rightarrow \)).

If \( p \) and \( q \) are statements, then \( \lnot p \) means not \( p \); \( \lnot q \) means not \( q \); \( p \land q \) means \( p \) and \( q \); \( p \lor q \) means \( p \) or \( q \); and \( p \rightarrow q \) means \( p \) implies \( q \). The operations are defined by truth tables. On the left below is the truth table for \( p \rightarrow q \). Notice that there are two possible conditions for \( p \) and \( q \), true \( (T) \) or false \( (F) \). If \( p \) is true, \( \lnot p \) is false; if \( p \) is false, \( \lnot p \) is true. Also shown are the truth tables for \( p \land q \), \( p \lor q \), and \( p \Rightarrow q \).

\[
\begin{array}{cccccccc}
 p & q & p & q & p \land q & p \lor q & p \Rightarrow q \\
 T & F & T & F & T & T & T \\
 T & T & T & T & T & T & T \\
 F & F & F & F & F & T & T \\
 F & T & F & T & T & T & T \\
 \end{array}
\]

You can use this information to find out under what conditions a complex statement is true.

Example Under what conditions is \( \lnot p \lor q \) true?

Create the truth table for the statement. Use the information from the truth table above for \( \lnot p \lor q \) to complete the last column.

\[
\begin{array}{cccccccc}
 p & q & \lnot p & \lor & q & \neg p & \lor & q \\
 T & T & F & T & T & T & T & T \\
 T & F & F & T & T & T & T & T \\
 F & T & T & T & T & T & T & T \\
 F & F & T & T & T & T & T & T \\
 \end{array}
\]

When one statement is true and one is false, the conjunction is true.

The truth table indicates that \( \lnot p \lor q \) is true in all cases except where \( p \) is true and \( q \) is false.

Use truth tables to determine the conditions under which each statement is true.

1. \( \lnot p \lor \lnot q \) all except where both \( p \) and \( q \) are true
2. \( \lnot p \rightarrow (p \lor q) \) all
3. \( (p \lor q) \lor (\lnot p \land \lnot q) \) all
4. \( (p \lor q) \lor (q \lor p) \) all
5. \( (\lnot p \land q) \lor (q \lor p) \) both \( p \) and \( q \) are true; both \( p \) and \( q \) are false
6. \( (\lnot p \land \lnot q) \lor (p \lor q) \) all