Chapter 2
Resource Masters
**Consumable Workbooks**  Many of the worksheets contained in the Chapter Resource Masters are available as consumable workbooks in both English and Spanish.

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<tr>
<td>Study Guide and Intervention Workbook</td>
<td>0-07-877344-X</td>
<td>978-0-07-877344-0</td>
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<td>Skills Practice Workbook</td>
<td>0-07-877346-6</td>
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<td>Practice Workbook</td>
<td>0-07-877347-4</td>
<td>978-0-07-877347-1</td>
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<td>Word Problem Practice Workbook</td>
<td>0-07-877349-0</td>
<td>978-0-07-877349-5</td>
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**Spanish Versions**

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<tr>
<td>Study Guide and Intervention Workbook</td>
<td>0-07-877345-8</td>
<td>978-0-07-877345-7</td>
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<tr>
<td>Practice Workbook</td>
<td>0-07-877348-2</td>
<td>978-0-07-877348-8</td>
</tr>
</tbody>
</table>

**Answers for Workbooks**  The answers for Chapter 2 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

**StudentWorks Plus™**  This CD-ROM includes the entire Student Edition test along with the English workbooks listed above.

**TeacherWorks Plus™**  All of the materials found in this booklet are included for viewing, printing, and editing in this CD-ROM.

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Teacher’s Guide to Using the 
Chapter 2 Resource Masters

The Chapter 2 Resource Masters includes the core materials needed for Chapter 2. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing on the TeacherWorks Plus™ CD-ROM.

Chapter Resources
Student-Built Glossary (pages 1–2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar. Give this to students before beginning Lesson 2-1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

Anticipation Guide (pages 3–4) This master, presented in both English and Spanish, is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources
Lesson Reading Guide Get Ready for the Lesson extends the discussion from the beginning of the Student Edition lesson. Read the Lesson asks students to interpret the context of and relationships among terms in the lesson. Finally, Remember What You Learned asks students to summarize what they have learned using various representation techniques. Use as a study tool for note taking or as an informal reading assignment. It is also a helpful tool for ELL (English Language Learners).

Study Guide and Intervention These masters provide vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

Practice This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

Word Problem Practice This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

Enrichment These activities may extend the concepts of the lesson, offer an historical or multicultural look at the concepts, or widen students’ perspectives on the mathematics they are learning. They are written for use with all levels of students.
Graphing Calculator, Scientific Calculator, or Spreadsheet Activities
These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.

Assessment Options
The assessment masters in the Chapter 2 Resource Masters offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

Student Recording Sheet This master corresponds with the standardized test practice at the end of the chapter.

Pre-AP Rubric This master provides information for teachers and students on how to assess performance on open-ended questions.

Quizzes Four free-response quizzes offer assessment at appropriate intervals in the chapter.

Mid-Chapter Test This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

Vocabulary Test This test is suitable for all students. It includes a list of vocabulary words and 10 questions to assess students’ knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

Leveled Chapter Tests
- **Form 1** contains multiple-choice questions and is intended for use with below grade level students.
- **Forms 2A and 2B** contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- **Forms 2C and 2D** contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- **Form 3** is a free-response test for use with above grade level students. All of the above mentioned tests include a free-response Bonus question.

Extended-Response Test Performance assessment tasks are suitable for all students. Sample answers and a scoring rubric are included for evaluation.

Standardized Test Practice These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

Answers
- The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters.
This is an alphabetical list of the key vocabulary terms you will learn in Chapter 2. As you study the chapter, complete each term’s definition or description. Remember to add the page number where you found the term. Add these pages to your Geometry Study Notebook to review vocabulary at the end of the chapter.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition/Description/Example</th>
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</thead>
<tbody>
<tr>
<td>conclusion</td>
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<tr>
<td>conditional statement</td>
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<td>conjecture</td>
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<td>conjunction</td>
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<td>contrapositive</td>
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<td>converse</td>
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<td>counterexample</td>
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<td>deductive argument</td>
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<td>deductive reasoning</td>
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<td>disjunction</td>
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<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition/Description/Example</th>
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<tr>
<td>hypothesis</td>
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<td>if-then statement</td>
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<td>truth value</td>
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<tr>
<td>two-column proof</td>
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</tbody>
</table>
Anticipation Guide

Reasoning and Proof

STEP 1

Before you begin Chapter 2

• Read each statement.
• Decide whether you Agree (A) or Disagree (D) with the statement.
• Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

<table>
<thead>
<tr>
<th>STEP 1 A, D, or NS</th>
<th>STEP 2 A or D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Inductive reasoning is reasoning that uses facts to reach logical conclusions.</td>
<td></td>
</tr>
<tr>
<td>2. A conjecture is an educated guess based on known information.</td>
<td></td>
</tr>
<tr>
<td>3. A conjunction is two statements joined by the word or.</td>
<td></td>
</tr>
<tr>
<td>4. A statement that can be written in if-then form is called a conditional statement.</td>
<td></td>
</tr>
<tr>
<td>5. Statements are logically equivalent if they both have the same truth values.</td>
<td></td>
</tr>
<tr>
<td>6. Deductive reasoning uses examples to make a conclusion.</td>
<td></td>
</tr>
<tr>
<td>7. A postulate is a mathematical statement that you must prove to be true.</td>
<td></td>
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<tr>
<td>8. A paragraph written to explain why a conjecture is true is called an informal proof.</td>
<td></td>
</tr>
<tr>
<td>9. Properties of Equality, postulates, and theorems can all be used to justify steps in a proof.</td>
<td></td>
</tr>
<tr>
<td>10. Once a statement has been proved, it can be used to prove other statements.</td>
<td></td>
</tr>
<tr>
<td>11. If two angles form a linear pair, then they are complimentary angles.</td>
<td></td>
</tr>
<tr>
<td>12. Congruence of angles is reflexive and symmetric, but not transitive.</td>
<td></td>
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</tbody>
</table>

STEP 2

After you complete Chapter 2

• Reread each statement and complete the last column by entering an A or a D.
• Did any of your opinions about the statements change from the first column?
• For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.
Ejercicios preparatorios
Razonamiento y prueba

Antes de comenzar el Capítulo 2

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a).

<table>
<thead>
<tr>
<th>PASO 1</th>
<th>Enunciado</th>
<th>PASO 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, D o NS</td>
<td>A o D</td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>El razonamiento inductivo usa hechos para sacar conclusiones lógicas.</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Una conjetura es una suposición informada que se basa en información conocida.</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Una conjunción son dos enunciados unidos por la palabra o.</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>Un enunciado que puede escribirse de la forma si-entonces, se llama enunciado condicional.</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Los enunciados son lógicamente equivalentes si ambos tienen los mismos valores verdaderos.</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>El razonamiento deductivo usa ejemplos para sacar una conclusión.</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>Un postulado es un enunciado matemático que debes probar para que sea verdadero.</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>Un postulado que se escribe para explicar por qué una conjetura es verdadera se llama prueba informal.</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>Las propiedades de igualdad, los postulados y los teoremas se pueden usar para justificar los pasos en una prueba.</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>Una vez que se prueba un enunciado, éste puede usarse para probar otros enunciados.</td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>Si dos ángulos forman un par lineal, entonces son ángulos complementarios.</td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td>La congruencia entre ángulos es reflexiva y simétrica, pero no transitiva.</td>
<td></td>
</tr>
</tbody>
</table>

Después de completar el Capítulo 2

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcase con una D.
Get Ready for the Lesson

Read the introduction to Lesson 2-1 in your textbook.

- How could people in the ancient Orient use inductive reasoning be used to assist in farming?

- Give an example of when you might use inductive reasoning in your daily life.

Read the Lesson

1. Explain in your own words the relationship between a conjecture, a counterexample, and inductive reasoning.

2. Make a conjecture about the next item in each sequence.
   
a. 5, 9, 13, 17
   b. 1, 1/3, 1/9, 1/27
   c. 0, 1, 3, 6, 10
   d. 8, 3, −2, −7
   e. 1, 8, 27, 64
   f. 1, −2, 4, −8
   g. 
   h. 

3. State whether each conjecture is true or false. If the conjecture is false, give a counterexample.
   a. The sum of two odd integers is even.
   b. The product of an odd integer and an even integer is odd.
   c. The opposite of an integer is a negative integer.
   d. The perfect squares (squares of whole numbers) alternate between odd and even.

Remember What You Learned

4. Write a short sentence that can help you remember why it only takes one counterexample to prove that a conjecture is false.
Make Conjectures  A conjecture is a guess based on analyzing information or observing a pattern. Making a conjecture after looking at several situations is called inductive reasoning.

Example 1  Make a conjecture about the next number in the sequence 1, 3, 9, 27, 81.

Analyze the numbers:
Notice that each number is a power of 3.

\[
\begin{array}{cccc}
1 & 3 & 9 & 27 & 81 \\
3^0 & 3^1 & 3^2 & 3^3 & 3^4 \\
\end{array}
\]

Conjecture: The next number will be \(3^5\) or 243.

Example 2  Make a conjecture about the number of small squares in the next figure.

Observe a pattern: The sides of the squares have measures 1, 2, and 3 units.

Conjecture: For the next figure, the side of the square will be 4 units, so the figure will have 16 small squares.

Exercises

Describe the pattern. Then make a conjecture about the next number in the sequence.

1. \(-5, 10, -20, 40\)

2. \(1, 10, 100, 1000\)

3. \(1, \frac{6}{5}, \frac{7}{5}, \frac{8}{5}\)

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.

4. \(A(-1, -1), B(2, 2), C(4, 4)\)

5. \(\angle 1 \) and \(\angle 2\) form a right angle.

6. \(\angle ABC \) and \(\angle DBE\) are vertical angles.

7. \(\angle E\) and \(\angle F\) are right angles.
Find Counterexamples A conjecture is false if there is even one situation in which the conjecture is not true. The false example is called a counterexample.

Example Determine whether the conjecture is true or false. If it is false, give a counterexample.

Given: \( AB \parallel BC \)

Conjecture: \( B \) is the midpoint of \( AB \).

Is it possible to draw a diagram with \( AB \parallel BC \) such that \( B \) is not the midpoint? This diagram is a counterexample because point \( B \) is not on \( AC \). The conjecture is false.

Exercises Determine whether each conjecture is true or false. Give a counterexample for any false conjecture.

1. Given: Points \( A \), \( B \), and \( C \) are collinear.
   \( \text{Conjecture: } AB + BC = AC \)

2. Given: \( \angle R \) and \( \angle S \) are supplementary.
   \( \angle R \) and \( \angle T \) are supplementary.
   \( \text{Conjecture: } \angle T \) and \( \angle S \) are congruent.

3. Given: \( \angle ABC \) and \( \angle DEF \) are supplementary.
   \( \text{Conjecture: } \angle ABC \) and \( \angle DEF \) form a linear pair.

4. Given: \( DE \perp EF \)
   \( \text{Conjecture: } \angle DEF \) is a right angle.
2-1 Skills Practice
Inductive Reasoning and Conjecture

Make a conjecture about the next item in each sequence.

1. □ □ □ □ □

2. −4, −1, 2, 5, 8
3. 6, \(\frac{11}{2}\), 5, \(\frac{9}{2}\), 4
4. −2, 4, −8, 16, −32

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.

5. Points A, B, and C are collinear, and D is between B and C.
6. Point P is the midpoint of \(\overline{NQ}\).

7. \(\angle 1, \angle 2, \angle 3, \text{ and } \angle 4\) form four linear pairs.
8. \(\angle 3 \equiv \angle 4\)

Determine whether each conjecture is true or false. Give a counterexample for any false conjecture.

9. Given: \(\angle ABC\) and \(\angle CBD\) form a linear pair.
   Conjecture: \(\angle ABC \equiv \angle CBD\)

10. Given: \(\overline{AB}, \overline{BC},\) and \(\overline{AC}\) are congruent.
    Conjecture: A, B, and C are collinear.

11. Given: \(AB + BC = AC\)
    Conjecture: \(AB = BC\)

12. Given: \(\angle 1\) is complementary to \(\angle 2\), and \(\angle 1\) is complementary to \(\angle 3\).
    Conjecture: \(\angle 2 \equiv \angle 3\)
2-1 Practice

Inductive Reasoning and Conjecture

Make a conjecture about the next item in each sequence.

1. 3, 6, 9, 12, 15, 18 ...

2. 5, −10, 15, −20

3. −2, 1, −2, 4, −1, 8

4. 12, 6, 3, 1.5, 0.75

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.

5. \( \angle ABC \) is a right angle.

6. Point \( S \) is between \( R \) and \( T \).

7. \( P, Q, R, \) and \( S \) are noncollinear and \( PQ \parallel QR \parallel RS \parallel SP \).

8. \( ABCD \) is a parallelogram.

Determine whether each conjecture is true or false. Give a counterexample for any false conjecture.

9. Given: \( S, T, \) and \( U \) are collinear and \( ST = TU \).
   Conjecture: \( T \) is the midpoint of \( SU \).

10. Given: \( \angle 1 \) and \( \angle 2 \) are adjacent angles.
    Conjecture: \( \angle 1 \) and \( \angle 2 \) form a linear pair.

11. Given: \( GH \) and \( JK \) form a right angle and intersect at \( P \).
    Conjecture: \( GH \perp JK \)

12. ALLERGIES Each spring, Rachel starts sneezing when the pear trees on her street blossom. She reasons that she is allergic to pear trees. Find a counterexample to Rachel’s conjecture.
1. **RAMPS** Rodney is rolling marbles down a ramp. Every second that passes, he measures how far the marbles travel. He records the information in the table shown below.

<table>
<thead>
<tr>
<th>Second</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (cm)</td>
<td>20</td>
<td>60</td>
<td>100</td>
<td>140</td>
</tr>
</tbody>
</table>

Make a conjecture about how far the marble will roll in the fifth second.

2. **PRIMES** A prime number is a number other than 1 that is divisible by only itself and 1. Lucille read that prime numbers are very important in cryptography, so she decided to find a systematic way of producing prime numbers. After some experimenting, she conjectured that $2^n - 1$ is a prime for all whole numbers $n > 1$. Find a counterexample to this conjecture.

3. **GENEALOGY** Miranda is developing a chart that shows her ancestry. She makes the three sketches shown below. The first dot represents herself. The second sketch represents herself and her parents. The third sketch represents herself, her parents, and her grandparents.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sketch what you think would be the next figure in the sequence.

4. **MEDALS** Barbara is in charge of the award medals for a sporting event. She has 31 medals to give out to various individuals on 6 competing teams. She asserts that at least one team will end up with more than 5 medals. Do you believe her assertion? If you do, try to explain why you think her assertion is true, and if you do not, explain how she can be wrong.

5. **PATTERNS** For Exercises 5–7, use the following information.

The figure shows a sequence of squares each made out of identical square tiles.

- 1
- 2
- 3

5. Starting from zero tiles, how many tiles do you need to make the first square? How many tiles do you have to add to the first square to get the second square? How many tiles do you have to add to the second square to get the third square?

6. Make a conjecture about the list of numbers you started writing in your answer to Exercise 5.

7. Make a conjecture about the sum of the first $n$ odd numbers.
Enrichment

2-1

Counterexamples

When you make a conclusion after examining several specific cases, you have used **inductive reasoning**. However, you must be cautious when using this form of reasoning. By finding only one counterexample, you disprove the conclusion.

**Example**

Is the statement \( \frac{1}{x} \leq 1 \) true when you replace \( x \) with 1, 2, and 3? Is the statement true for all reals? If possible, find a counterexample.

\[ \frac{1}{1} = 1, \frac{1}{2} < 1, \text{ and } \frac{1}{3} < 1. \] But when \( x = \frac{1}{2} \), then \( \frac{1}{x} = 2 \). This counterexample shows that the statement is not always true.

**Exercises**

1. The coldest day of the year in Chicago occurred in January for five straight years. Is it safe to conclude that the coldest day in Chicago is always in January?

2. Suppose John misses the school bus four Tuesdays in a row. Can you safely conclude that John misses the school bus every Tuesday?

3. Is the equation \( \sqrt{k^2} = k \) true when you replace \( k \) with 1, 2, and 3? Is the equation true for all integers? If possible, find a counterexample.

4. Is the statement \( 2x = x + x \) true when you replace \( x \) with \( \frac{1}{2}, 4 \), and 0.7? Is the statement true for all real numbers? If possible, find a counterexample.

5. Suppose you draw four points \( A, B, C, \) and \( D \) and then draw \( AB, BC, CD, \) and \( DA \). Does this procedure give a quadrilateral always or only sometimes? Explain your answers with figures.

6. Suppose you draw a circle, mark three points on it, and connect them. Will the angles of the triangle be acute? Explain your answers with figures.
Graphing Calculator Activity

Conjectures

You can use a graphing calculator to illustrate conjectures you make about a group of points described by ordered pairs of numbers.

**Example**

Write a conjecture based on the given information. Use a graphing calculator to illustrate your conjecture.

**Given:** X(20.5, 41.5), Y(11, 22.5), Z(−10.2, −19.9)

**Conjecture:** X, Y, and Z are collinear.

To illustrate this conjecture, plot the points X, Y, and Z.

**Step 1**
To plot X, press 2nd [DRAW] ▶. Select 1: Pt-On from the menu. Press 20.5 ▼ 41.5 ENTER.

**Step 2**
To plot Y, press 2nd [QUIT] 2nd [DRAW] ▶. Select 1: Pt-On from the menu. Press 11 ▼ 22.5 ENTER.

**Step 3**
To plot Z, press 2nd [QUIT] 2nd [DRAW] ▶. Select 1: Pt-On from the menu. Press (−) 10.2 ▼ (−) 19.9 ENTER.

**Step 4**
To verify that the points are collinear, press 2nd [QUIT] 2nd [DRAW]. Select 2: Line from the menu. Draw XZ by pressing 20.5 ▼ 41.5 ▼ (−) 10.2 ▼ (−) 19.9 ENTER.

The segment drawn shows the points to be collinear.

**Exercises**

Write a conjecture based on the given information. Use a graphing calculator to draw a figure to illustrate your conjecture.

1. Given: M(16, 48.6), N(9.2, 28.2), P(−12.4, −36.6)

2. Given: A(15, 19), B(15, 25), C(21, 19), D(21, 25)

3. Given: R(−14, 10), S(−14, −15), T(25, −15)
Lesson Reading Guide

Logic

Get Ready for the Lesson

Read the introduction to Lesson 2-2 in your textbook.

How can you use logic to help you answer a multiple-choice question on a standardized test if you are not sure of the correct answer?

Read the Lesson

1. Supply one or two words to complete each sentence.
   a. Two or more statements can be joined to form a __________ statement.
   b. A statement that is formed by joining two statements with the word or is called a __________.
   c. The truth or falsity of a statement is called its __________.
   d. A statement that is formed by joining two statements with the word and is called a __________.
   e. A statement that has the opposite truth value and the opposite meaning from a given statement is called the __________ of the statement.

2. Use true or false to complete each sentence.
   a. If a statement is true, then its negation is __________.
   b. If a statement is false, then its negation is __________.
   c. If two statements are both true, then their conjunction is __________ and their disjunction is __________.
   d. If two statements are both false, then their conjunction is __________ and their disjunction is __________.
   e. If one statement is true and another is false, then their conjunction is __________ and their disjunction is __________.

3. Consider the following statements:
   p: Chicago is the capital of Illinois.
   q: Sacramento is the capital of California.

   Write each statement symbolically and then find its truth value.
   a. Sacramento is not the capital of California.
   b. Sacramento is the capital of California and Chicago is not the capital of Illinois.

Remember What You Learned

4. Prefixes can often help you to remember the meaning of words or to distinguish between similar words. Use your dictionary to find the meanings of the prefixes con and dis and explain how these meanings can help you remember the difference between a conjunction and a disjunction.
Determine Truth Values  A statement is any sentence that is either true or false. The truth or falsity of a statement is its truth value. A statement can be represented by using a letter. For example, 

Statement \( p \): Chicago is a city in Illinois. The truth value of statement \( p \) is true.

Several statements can be joined in a compound statement.

<table>
<thead>
<tr>
<th>Statement ( p ) and statement ( q ) joined by the word and is a conjunction.</th>
<th>Statement ( p ) and statement ( q ) joined by the word or is a disjunction.</th>
<th>Negation: ( \sim p ) is the negation of the statement ( p ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols: ( p \wedge q ) (Read: ( p ) and ( q ))</td>
<td>Symbols: ( p \vee q ) (Read: ( p ) or ( q ))</td>
<td>Symbols: ( \sim p ) (Read: not ( p ))</td>
</tr>
<tr>
<td>The conjunction ( p \wedge q ) is true only when both ( p ) and ( q ) are true.</td>
<td>The disjunction ( p \vee q ) is true if ( p ) is true, if ( q ) is true, or if both are true.</td>
<td>The statements ( p ) and ( \sim p ) have opposite truth values.</td>
</tr>
</tbody>
</table>

Example 1  Write a compound statement for each conjunction. Then find its truth value.

- \( p \): An elephant is a mammal.
- \( q \): A square has four right angles.

\[ p \wedge q \]

Join the statements with and: An elephant is a mammal and a square has four right angles. Both parts of the statement are true so the compound statement is true.

\[ \sim p \wedge q \]

\( \sim p \) is the statement “An elephant is not a mammal.” Join \( \sim p \) and \( q \) with the word and: An elephant is not a mammal and a square has four right angles. The first part of the compound statement, \( \sim p \), is false. Therefore the compound statement is false.

Example 2  Write a compound statement for each disjunction. Then find its truth value.

- \( p \): A diameter of a circle is twice the radius.
- \( q \): A rectangle has four equal sides.

\[ p \vee q \]

Join the statements \( p \) and \( q \) with the word or: A diameter of a circle is twice the radius or a rectangle has four equal sides. The first part of the compound statement, \( p \), is true, so the compound statement is true.

\[ \sim p \vee q \]

Join \( \sim p \) and \( q \) with the word or: A diameter of a circle is not twice the radius or a rectangle has four equal sides. Neither part of the disjunction is true, so the compound statement is false.

Exercises

Write a compound statement for each conjunction and disjunction. Then find its truth value.

\( p: 10 + 8 = 18 \quad q: \) September has 30 days. \( r: \) A rectangle has four sides.

1. \( p \) and \( q \)
2. \( p \) or \( r \)
3. \( q \) or \( r \)
4. \( q \) and \( \sim r \)
**2-2 Study Guide and Intervention (continued)**

**Logic**

**Truth Tables** One way to organize the truth values of statements is in a **truth table**. The truth tables for negation, conjunction, and disjunction are shown at the right.

<table>
<thead>
<tr>
<th>Negation</th>
<th>Conjunction</th>
<th>Disjunction</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>( \sim p )</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p \land q )</th>
<th>( p \lor q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

**Example 1** Construct a truth table for the compound statement \( q \lor r \). Use the disjunction table.

<table>
<thead>
<tr>
<th>( q )</th>
<th>( r )</th>
<th>( q \lor r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

**Example 2** Construct a truth table for the compound statement \( p \land (q \lor r) \).

Use the disjunction table for \( (q \lor r) \). Then use the conjunction table for \( p \) and \( (q \lor r) \).

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( r )</th>
<th>( q \lor r )</th>
<th>( p \land (q \lor r) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
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<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
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<tr>
<td>F</td>
<td>T</td>
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<td>F</td>
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<tr>
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<td>T</td>
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<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

**Exercises**

Construct a truth table for each compound statement.

1. \( p \lor r \)  
2. \( \neg p \lor q \)  
3. \( q \land \neg r \)  
4. \( \neg p \land \neg r \)  
5. \( (p \land r) \lor q \)
Skills Practice

Logic

Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value.

\( p: -3 - 2 = -5 \)
\( q: \text{Vertical angles are congruent.} \)
\( r: 2 + 8 > 10 \)
\( s: \text{The sum of the measures of complementary angles is } 90^\circ. \)

1. \( p \) and \( q \)

2. \( p \land r \)

3. \( p \) or \( s \)

4. \( r \lor s \)

5. \( p \land \sim q \)

6. \( q \lor \sim r \)

Copy and complete each truth table.

7. 

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( \sim p )</th>
<th>( \sim p \land q )</th>
<th>( \sim (\sim p \land q) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
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<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

8. 

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( \sim q )</th>
<th>( p \lor \sim q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Construct a truth table for each compound statement.

9. \( \sim q \land r \)

10. \( \sim p \lor \sim r \)
Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value.

\( p: 60 \text{ seconds} = 1 \text{ minute} \)
\( q: \text{Congruent supplementary angles each have a measure of 90.} \)
\( r: -12 + 11 < -1 \)

1. \( p \land q \)
2. \( q \lor r \)
3. \( \neg p \lor q \)
4. \( \neg p \land \neg r \)

Copy and complete each truth table.

5. \[
\begin{array}{c|c|c|c|c|c}
 p & q & \neg p & \neg q & \neg p \lor \neg q \\
 T & T & & & \\
 T & F & & & \\
 F & T & & & \\
 F & F & & & \\
\end{array}
\]

6. \[
\begin{array}{c|c|c|c|c|c}
 p & q & \neg p & \neg q & \neg p \lor \neg q & p \land (\neg p \lor q) \\
 T & T & & & \\
 T & F & & & \\
 F & T & & & \\
 F & F & & & \\
\end{array}
\]

Construct a truth table for each compound statement.

7. \( q \lor (p \land \neg q) \)
8. \( \neg q \land (\neg p \lor q) \)

**SCHOOL** For Exercises 9 and 10, use the following information.

The Venn diagram shows the number of students in the band who work after school or on the weekends.

9. How many students work after school and on weekends?

10. How many students work after school or on weekends?
Word Problem Practice

Logic

1. **HOCKEY** Carol asked John if his hockey team won the game last night and if he scored a goal. John said “yes.” Carol then asked Peter if he or John scored a goal at the game. Peter said “yes.” What can you conclude about whether or not Peter scored?

2. **CHOCOLATE** Nash has a bag of miniature chocolate bars that come in two distinct types: dark and milk. Nash picks a chocolate out of the bag. Consider these statements:

   \( p \): the chocolate bar is dark chocolate

   \( q \): the chocolate bar is milk chocolate

   Is the following statement true?

   \[ \sim (\sim p \land \sim q) \]

3. **VIDEO GAMES** Harold is allowed to play video games only if he washes the dishes or takes out the trash. However, if Harold does not do his homework, he is not allowed to play video games under any circumstance. Complete the truth table.

   \( p \): Harold has washed the dishes

   \( q \): Harold has taken out the trash

   \( r \): Harold has done his homework

   \( s \): Harold is allowed to play video games

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( r )</th>
<th>( s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
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<tr>
<td>T</td>
<td>T</td>
<td>F</td>
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<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

4. **CIRCUITS** In Earl’s house, the dining room light is controlled by two switches according to the following table.

<table>
<thead>
<tr>
<th>Switch A</th>
<th>Switch B</th>
<th>Light</th>
</tr>
</thead>
<tbody>
<tr>
<td>up</td>
<td>up</td>
<td>off</td>
</tr>
<tr>
<td>up</td>
<td>down</td>
<td>on</td>
</tr>
<tr>
<td>down</td>
<td>up</td>
<td>on</td>
</tr>
<tr>
<td>down</td>
<td>down</td>
<td>off</td>
</tr>
</tbody>
</table>

   If up and on are considered true and down and off are considered false, write an expression that gives the truth value of the light as a function of the truth values of the two switches.

Reading

For Exercises 5–7, use the following information.

Two hundred people were asked what kind of literature they like to read. They could choose among novels, poetry, and plays. The results are shown in the Venn diagram.

5. How many people said they like all three types of literature?

6. How many like to read poetry?

7. What percentage of the people who like plays also like novels and poetry?


Sudoku

Sudoku is a math puzzle that requires logic to solve. A Sudoku puzzle is typically a $9 \times 9$ grid with each square subdivided into nine $3 \times 3$ squares. The puzzle starts with some of the numbers given and the goal is to fill in the rest using the following rules.

- Each row and each column has every number, 1 through 9, with none repeated.
- Each $3 \times 3$ grid must contain every number, 1 through 9, with none repeated.


Exercises

1. What is a good starting point? Why?

2. Explain how you can use the second rule to have all the numbers to solve the larger puzzle.

3. Complete the puzzle.
Spreadsheet Activity

Truth Tables

You can use a spreadsheet to create truth tables.

Example

Use a spreadsheet to complete the following truth table.

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>q</td>
<td>~p</td>
<td>~q</td>
<td>p and q</td>
<td>p or q</td>
<td>~p or ~q</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step 1 Enter the title for each column in row 1.
Step 2 In cells A2, A3, A4, and A5, enter T, T, F, F. Press ENTER after each entry.
Step 3 In cells B2, B3, B4, and B5, enter T, F, T, F. Press ENTER after each entry.
Step 4 In cell C2, enter an equals sign followed by IF(A2="T", "F", "T"). This will return ~p. Click on the bottom right corner of cell C2 and drag it to cell C5 to return all four entries.
Step 5 In cell D2, enter an equals sign followed by IF(B2="T", "F", "T"). This will return ~q. Click on the bottom right corner of cell D2 and drag it to cell D5 to return all four entries.
Step 6 In cell E2, enter an equals sign followed by AND(A2="T", B2="T"). This will return p and q. Click on the bottom right corner of cell E2 and drag it to cell E5 to return all four entries.
Step 7 In cell F2, enter an equals sign followed by OR(A2="T", B2="T"). This will return p or q. Click on the bottom right corner of cell F2 and drag it to cell F5 to return all four entries.
Step 8 In cell G2, enter an equals sign followed by OR(C2="T", D2="T"). This will return ~p or ~q. Click on the bottom right corner of cell G2 and drag it to cell G5 to return all four entries.

Exercises

Use a spreadsheet to complete the following truth table.

<p>| | | | | | | |</p>
<table>
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Get Ready for the Lesson

Read the introduction to Lesson 2-3 in your textbook.

Does the second advertising statement in the introduction mean that you will not get a free phone if you sign a contract for only six months of service? Explain your answer.

Read the Lesson

1. Identify the hypothesis and conclusion of each statement.
   a. If you are a registered voter, then you are at least 18 years old.
   b. If two integers are even, their product is even.

2. Complete each sentence.
   a. The statement that is formed by replacing both the hypothesis and the conclusion of a conditional with their negations is the ________________.
   b. The statement that is formed by exchanging the hypothesis and conclusion of a conditional is the ________________.

3. Consider the following statement:
   You live in North America if you live in the United States.
   a. Write this conditional statement in if-then form and give its truth value. If the statement is false, give a counterexample.
   b. Write the inverse of the given conditional statement in if-then form and give its truth value. If the statement is false, give a counterexample.
   c. Write the contrapositive of the given conditional statement in if-then form and give its truth value. If the statement is false, give a counterexample.
   d. Write the converse of the given conditional statement in if-then form and give its truth value. If the statement is false, give a counterexample.

Remember What You Learned

4. When working with a conditional statement and its three related conditionals, what is an easy way to remember which statements are logically equivalent to each other?
Conditional Statements

If-then Statements  An if-then statement is a statement such as “If you are reading this page, then you are studying math.” A statement that can be written in if-then form is called a conditional statement. The phrase immediately following the word if is the hypothesis. The phrase immediately following the word then is the conclusion.

A conditional statement can be represented in symbols as $p \rightarrow q$, which is read “$p$ implies $q$” or “if $p$, then $q$.”

Example 1  Identify the hypothesis and conclusion of the statement.

If $\angle X \cong \angle R$ and $\angle R \cong \angle S$, then $\angle X \cong \angle S$.

Example 2  Identify the hypothesis and conclusion. Write the statement in if-then form.

You receive a free pizza with 12 coupons.
If you have 12 coupons, then you receive a free pizza.

Exercises

Identify the hypothesis and conclusion of each statement.

1. If it is Saturday, then there is no school.
2. If $x - 8 = 32$, then $x = 40$.
3. If a polygon has four right angles, then the polygon is a rectangle.

Write each statement in if-then form.

4. All apes love bananas.
5. The sum of the measures of complementary angles is 90.
6. Collinear points lie on the same line.

Determine the truth value of the following statement for each set of conditions.

If it does not rain this Saturday, we will have a picnic.

7. It rains this Saturday, and we have a picnic.
8. It rains this Saturday, and we don’t have a picnic.
9. It doesn’t rain this Saturday, and we have a picnic.
10. It doesn’t rain this Saturday, and we don’t have a picnic.
Converse, Inverse, and Contrapositive  If you change the hypothesis or conclusion of a conditional statement, you form a related conditional. This chart shows the three related conditionals, converse, inverse, and contrapositive, and how they are related to a conditional statement.

<table>
<thead>
<tr>
<th></th>
<th>Symbols</th>
<th>Formed by</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditional</td>
<td>$p \rightarrow q$</td>
<td>using the given hypothesis and conclusion</td>
<td>If two angles are vertical angles, then they are congruent.</td>
</tr>
<tr>
<td>Converse</td>
<td>$q \rightarrow p$</td>
<td>exchanging the hypothesis and conclusion</td>
<td>If two angles are congruent, then they are vertical angles.</td>
</tr>
<tr>
<td>Inverse</td>
<td>$\sim p \rightarrow \sim q$</td>
<td>replacing the hypothesis with its negation and replacing the conclusion with its negation</td>
<td>If two angles are not vertical angles, then they are not congruent.</td>
</tr>
<tr>
<td>Contrapositive</td>
<td>$\sim q \rightarrow \sim p$</td>
<td>negating the hypothesis, negating the conclusion, and switching them</td>
<td>If two angles are not congruent, then they are not vertical angles.</td>
</tr>
</tbody>
</table>

Just as a conditional statement can be true or false, the related conditionals also can be true or false. A conditional statement always has the same truth value as its contrapositive, and the converse and inverse always have the same truth value.

Exercises

Write the converse, inverse, and contrapositive of each conditional statement. Tell which statements are true and which statements are false.

1. If you live in San Diego, then you live in California.

2. If a polygon is a rectangle, then it is a square.

3. If two angles are complementary, then the sum of their measures is 90.
Identify the hypothesis and conclusion of each statement.

1. If you purchase a computer and do not like it, then you can return it within 30 days.

2. If $x + 8 = 4$, then $x = -4$.

3. If the drama class raises $2000, then they will go on tour.

Write each statement in if-then form.

4. A polygon with four sides is a quadrilateral.


6. An acute angle has a measure less than 90.

Determine the truth value of the following statement for each set of conditions.

If you finish your homework by 5 P.M., then you go out to dinner.

7. You finish your homework by 5 P.M. and you go out to dinner.

8. You finish your homework by 4 P.M. and you go out to dinner.

9. You finish your homework by 5 P.M. and you do not go out to dinner.

10. Write the converse, inverse, and contrapositive of the conditional statement. Determine whether each statement is true or false. If a statement is false, find a counterexample.

If 89 is divisible by 2, then 89 is an even number.
2-3 Practice

**Conditional Statements**

Identify the hypothesis and conclusion of each statement.

1. If $3x + 4 = -5$, then $x = -3$.

2. If you take a class in television broadcasting, then you will film a sporting event.

Write each statement in if-then form.

3. “Those who do not remember the past are condemned to repeat it.” *George Santayana*

4. Adjacent angles share a common vertex and a common side.

Determine the truth value of the following statement for each set of conditions.

*If DVD players are on sale for less than $100, then you buy one.*

5. DVD players are on sale for $95 and you buy one.

6. DVD players are on sale for $100 and you do not buy one.

7. DVD players are not on sale for under $100 and you do not buy one.

8. Write the converse, inverse, and contrapositive of the conditional statement. Determine whether each statement is true or false. If a statement is false, find a counterexample.

*If $( -8 )^2 > 0$, then $-8 > 0$.*

**SUMMER CAMP** For Exercises 9 and 10, use the following information.

Older campers who attend Woodland Falls Camp are expected to work. Campers who are juniors wait on tables.

9. Write a conditional statement in if-then form.

10. Write the converse of your conditional statement.
1. **TANNING** Maya reads in a paper that people who tan themselves under the Sun for extended periods are at increased risk of skin cancer. From this information, can she conclude that she will not increase her risk of skin cancer if she avoids tanning for extended periods of time?

2. **PARALLELOGRAMS** Clark says that being a parallelogram is equivalent to being a quadrilateral with equal opposite angles. Write his statement in if-then form.

3. **AIR TRAVEL** Ulma is waiting to board an airplane. Over the speakers she hears a flight attendant say “If you are seated in rows 10 to 20, you may now board.” What are the inverse, converse, and the contrapositive of this statement?

4. **MEDICATION** Linda’s medicine bottle says “If you are pregnant, then you cannot take this medicine.” What are the inverse, converse, and the contrapositive of this statement?

---

**VENN DIAGRAMS** For Exercises 5–8, use the following information.

Jose made this Venn diagram to show how rectangles, squares, and rhombi are related. (A rhombus is a quadrilateral with four sides of equal length.)

Let Q be a quadrilateral. For each problem tell whether the statement is true or false. If it is false, provide a counterexample.

5. If Q is a square, then Q a rectangle.

6. If Q is not a rectangle, then Q is not a rhombus.

7. If Q is a rectangle but not a square, then Q is not a rhombus.

8. If Q is not a rhombus, then Q is not a square.
Venn Diagrams

A type of drawing called a **Venn diagram** can be useful in explaining conditional statements. A Venn diagram uses circles to represent sets of objects.

Consider the statement “All rabbits have long ears.” To make a Venn diagram for this statement, a large circle is drawn to represent all animals with long ears. Then a smaller circle is drawn inside the first to represent all rabbits. The Venn diagram shows that every rabbit is included in the group of long-eared animals.

The set of rabbits is called a **subset** of the set of long-eared animals.

The Venn diagram can also explain how to write the statement, “All rabbits have long ears,” in if-then form. Every rabbit is in the group of long-eared animals, so if an animal is a rabbit, then it has long ears.

For each statement, draw a Venn diagram. Then write the sentence in if-then form.

1. Every dog has long hair.
2. All rational numbers are real.
3. People who live in Iowa like corn.
4. Staff members are allowed in the faculty lounge.
Get Ready for the Lesson

Read the introduction to Lesson 2-4 in your textbook.

Suppose a doctor wants to use the dose chart in your textbook to prescribe an antibiotic, but the only scale in her office gives weights in pounds. How can she use the fact that 1 kilogram is about 2.2 pounds to determine the correct dose for a patient?

Read the Lesson

If \( s \), \( t \), and \( u \) are three statements, match each description from the list on the left with a symbolic statement from the list on the right.

1. negation of \( t \)    a. \( s \lor u \)
2. conjunction of \( s \) and \( u \)    b. \( [(s \rightarrow t) \land s] \rightarrow t \)
3. converse of \( s \rightarrow t \)    c. \( \neg s \rightarrow \neg u \)
4. disjunction of \( s \) and \( u \)    d. \( \neg u \rightarrow \neg s \)
5. Law of Detachment
6. contrapositive of \( s \rightarrow t \)    f. \( [(u \rightarrow t) \land (t \rightarrow s)] \rightarrow (u \rightarrow s) \)
7. inverse of \( s \rightarrow u \)    g. \( s \land u \)
8. contrapositive of \( s \rightarrow u \)    h. \( t \rightarrow s \)
9. Law of Syllogism
10. negation of \( \neg t \)    i. \( t \)
11. negation of \( \neg t \)    j. \( \neg t \rightarrow \neg s \)

11. Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write invalid.

a. (1) Every square is a parallelogram.
   (2) Every parallelogram is a polygon.
   (3) Every square is a polygon.

b. (1) If two lines that lie in the same plane do not intersect, they are parallel.
   (2) Lines \( \ell \) and \( m \) lie in plane \( \ell \) and do not intersect.
   (3) Lines \( \ell \) and \( m \) are parallel.

c. (1) Perpendicular lines intersect to form four right angles.
   (2) \( \angle A \), \( \angle B \), \( \angle C \), and \( \angle D \) are four right angles.
   (3) \( \angle A \), \( \angle B \), \( \angle C \), and \( \angle D \) are formed by intersecting perpendicular lines.

Remember What You Learned

12. A good way to remember something is to explain it to someone else. Suppose that a classmate is having trouble remembering what the Law of Detachment means?
2-4 Study Guide and Intervention

Deductive Reasoning

Law of Detachment Deductive reasoning is the process of using facts, rules, definitions, or properties to reach conclusions. One form of deductive reasoning that draws conclusions from a true conditional \( p \rightarrow q \) and a true statement \( p \) is called the Law of Detachment.

<table>
<thead>
<tr>
<th>Law of Detachment</th>
<th>If ( p \rightarrow q ) is true and ( p ) is true, then ( q ) is true.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td>( [(p \rightarrow q) \land p] \rightarrow q )</td>
</tr>
</tbody>
</table>

Example The statement If two angles are supplementary to the same angle, then they are congruent is a true conditional. Determine whether each conclusion is valid based on the given information. Explain your reasoning.

a. Given: \( \angle A \) and \( \angle C \) are supplementary to \( \angle B \).
   Conclusion: \( \angle A \) is congruent to \( \angle C \).
   The statement \( \angle A \) and \( \angle C \) are supplementary to \( \angle B \) is the hypothesis of the conditional. Therefore, by the Law of Detachment, the conclusion is true.

b. Given: \( \angle A \) is congruent to \( \angle C \).
   Conclusion: \( \angle A \) and \( \angle C \) are supplementary to \( \angle B \).
   The statement \( \angle A \) is congruent to \( \angle C \) is not the hypothesis of the conditional, so the Law of Detachment cannot be used. The conclusion is not valid.

Exercise Determine whether each conclusion is valid based on the true conditional given. If not, write invalid. Explain your reasoning.

If two angles are complementary to the same angle, then the angles are congruent.

1. Given: \( \angle A \) and \( \angle C \) are complementary to \( \angle B \).
   Conclusion: \( \angle A \) is congruent to \( \angle C \).

2. Given: \( \angle A \equiv \angle C \)
   Conclusion: \( \angle A \) and \( \angle C \) are complements of \( \angle B \).

3. Given: \( \angle E \) and \( \angle F \) are complementary to \( \angle G \).
   Conclusion: \( \angle E \) and \( \angle F \) are vertical angles.
Chapter 2

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Glencoe Geometry

Study Guide and Intervention (continued)

Deductive Reasoning

Law of Syllogism Another way to make a valid conclusion is to use the Law of Syllogism. It is similar to the Transitive Property.

<table>
<thead>
<tr>
<th>Law of Syllogism</th>
<th>If ( p \rightarrow q ) is true and ( q \rightarrow r ) is true, then ( p \rightarrow r ) is also true.</th>
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</thead>
<tbody>
<tr>
<td>Symbols</td>
<td>([ (p \rightarrow q) \land (q \rightarrow r) ] \rightarrow (p \rightarrow r) )</td>
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</table>

**Example** The two conditional statements below are true. Use the Law of Syllogism to find a valid conclusion. State the conclusion.

(1) If a number is a whole number, then the number is an integer.
(2) If a number is an integer, then it is a rational number.

\( p \): A number is a whole number.
\( q \): A number is an integer.
\( r \): A number is a rational number.

The two conditional statements are \( p \rightarrow q \) and \( q \rightarrow r \). Using the Law of Syllogism, a valid conclusion is \( p \rightarrow r \). A statement of \( p \rightarrow r \) is “if a number is a whole number, then it is a rational number.”

**Exercises**

Determine whether you can use the Law of Syllogism to reach a valid conclusion from each set of statements.

1. If a dog eats Superdog Dog Food, he will be happy.
   Rover is happy.

2. If an angle is supplementary to an obtuse angle, then it is acute.
   If an angle is acute, then its measure is less than 90.

3. If the measure of \( \angle A \) is less than 90, then \( \angle A \) is acute.
   If \( \angle A \) is acute, then \( \angle A \equiv \angle B \).

4. If an angle is a right angle, then the measure of the angle is 90.
   If two lines are perpendicular, then they form a right angle.

5. If you study for the test, then you will receive a high grade.
   Your grade on the test is high.
**Deductive Reasoning**

Determine whether the stated conclusion is valid based on the given information. If not, write invalid. Explain your reasoning.

*If the sum of the measures of two angles is 180, then the angles are supplementary.*

1. Given: \( m\angle A + m\angle B \) is 180.
   Conclusion: \( \angle A \) and \( \angle B \) are supplementary.

2. Given: \( m\angle ABC \) is 95 and \( m\angle DEF \) is 90.
   Conclusion: \( \angle ABC \) and \( \angle DEF \) are supplementary.

3. Given: \( \angle 1 \) and \( \angle 2 \) are a linear pair.
   Conclusion: \( \angle 1 \) and \( \angle 2 \) are supplementary.

Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements. If a valid conclusion is possible, write it.

4. If two angles are complementary, then the sum of their measures is 90.
   If the sum of the measures of two angles is 90, then both of the angles are acute.

5. If the heat wave continues, then air conditioning will be used more frequently.
   If air conditioning is used more frequently, then energy costs will be higher.

Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write invalid.

6. (1) If it is Tuesday, then Marla tutors chemistry.
   (2) If Marla tutors chemistry, then she arrives home at 4 P.M.
   (3) If Marla arrives at home at 4 P.M., then it is Tuesday.

7. (1) If a marine animal is a starfish, then it lives in the intertidal zone of the ocean.
   (2) The intertidal zone is the least stable of the ocean zones.
   (3) If a marine animal is a starfish, then it lives in the least stable of the ocean zones.
Determine whether the stated conclusion is valid based on the given information. If not, write invalid. Explain your reasoning.

If a point is the midpoint of a segment, then it divides the segment into two congruent segments.

1. Given: \( R \) is the midpoint of \( QS \).
   Conclusion: \( QR \cong RS \)

2. Given: \( AB \cong BC \)
   Conclusion: \( B \) divides \( AC \) into two congruent segments.

Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements. If a valid conclusion is possible, write it.

3. If two angles form a linear pair, then the two angles are supplementary.
   If two angles are supplementary, then the sum of their measures is 180.

4. If a hurricane is Category 5, then winds are greater than 155 miles per hour.
   If winds are greater than 155 miles per hour, then trees, shrubs, and signs are blown down.

Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write invalid.

5. (1) If a whole number is even, then its square is divisible by 4.
   (2) The number I am thinking of is an even whole number.
   (3) The square of the number I am thinking of is divisible by 4.

6. (1) If the football team wins its homecoming game, then Conrad will attend the school dance the following Friday.
   (2) Conrad attends the school dance on Friday.
   (3) The football team won the homecoming game.

7. BIOLOGY If an organism is a parasite, then it survives by living on or in a host organism. If a parasite lives in or on a host organism, then it harms its host. What conclusion can you draw if a virus is a parasite?
Word Problem Practice  
**Deductive Reasoning**

1. **SIGNS** Two signs are posted on a haunted house.

![Signs](image)

Inside the haunted house, you find a child with his parent. What can you deduce about the age of the child based on the house rules?

2. **LOGIC** As Laura's mother rushed off to work, she quickly gave Laura some instructions. “If you need me, try my cell . . . if I don’t answer it means I’m in a meeting, but don’t worry, the meeting won’t last more than 30 minutes and I’ll call you back when it’s over.” Later that day, Laura needed her mother, but her mother was stuck in a meeting and couldn’t answer the phone. Laura concludes that she will have to wait no more than 30 minutes before she gets a call back from her mother. What law of logic did Laura use to draw this conclusion?

3. **MUSIC** Composer Ludwig van Beethoven wrote 9 symphonies and 5 piano concertos. If you lived in Vienna in the early 1800s, you could attend a concert conducted by Beethoven himself. Write a valid conclusion to the hypothesis *If Mozart could not attend a concert conducted by Beethoven, . . .*

4. **DIRECTIONS** Hank has an appointment to see a financial advisor on the fifteenth floor of an office building. When he gets to the building, the people at the front desk tell him that if he wants to go to the fifteenth floor, then he must take the red elevator. While looking for the red elevator, a guard informs him that if he wants to find the red elevator he must find the replica of Michelangelo’s David. When he finally got to the fifteenth floor, his financial advisor greeted him asking, “What did you think of the Michelangelo?” How did Hank’s financial advisor conclude that Hank must have seen the Michelangelo statue?

5. **LAWS** For Exercises 5 and 6, use the following information.

The law says that if you are under 21, then you are not allowed to drink alcoholic beverages and if you are under 18, then you are not allowed to vote. For each problem give the possible ages of the person described or state that the person cannot exist.

5. John cannot drink wine legally but is allowed to vote.

6. Mary cannot vote legally but can drink beer legally.
Valid and Faulty Arguments

Consider the statements at the right. What conclusions can you make?

From statements 1 and 3, it is correct to conclude that Boots purrs if it is happy. However, it is faulty to conclude from only statements 2 and 3 that Boots is happy. The if-then form of statement 3 is If a cat is happy, then it purrs.

Advertisers often use faulty logic in subtle ways to help sell their products. By studying the arguments, you can decide whether the argument is valid or faulty.

Decide if each argument is valid or faulty.

1. (1) If you buy Tuff Cote luggage, it will survive airline travel. (2) Justin buys Tuff Cote luggage. Conclusion: Justin’s luggage will survive airline travel.

2. (1) If you buy Tuff Cote luggage, it will survive airline travel. (2) Justin’s luggage survived airline travel. Conclusion: Justin has Tuff Cote luggage.

3. (1) If you use Clear Line long distance service, you will have clear reception. (2) Anna has clear long distance reception. Conclusion: Anna uses Clear Line long distance service.

4. (1) If you read the book Beautiful Braids, you will be able to make beautiful braids easily. (2) Nancy read the book Beautiful Braids. Conclusion: Nancy can make beautiful braids easily.

5. (1) If you buy a word processor, you will be able to write letters faster. (2) Tania bought a word processor. Conclusion: Tania will be able to write letters faster.

6. (1) Great swimmers wear AquaLine swimwear. (2) Gina wears AquaLine swimwear. Conclusion: Gina is a great swimmer.

7. Write an example of faulty logic that you have seen in an advertisement.
Lesson Reading Guide

Postulates and Paragraph Proofs

Get Ready for the Lesson

Read the introduction to Lesson 2-5 in your textbook.

Postulates are often described as statements that are so basic and so clearly correct that people will be willing to accept them as true without asking for evidence or proof. Give a statement about numbers that you think most people would accept as true without evidence.

Read the Lesson

1. Determine whether each of the following is a correct or incorrect statement of a geometric postulate. If the statement is incorrect, replace the underlined words to make the statement correct.
   a. A plane contains at least two points that do not lie on the same line.
   b. If two planes intersect, then the intersection is a line.
   c. Through any four points not on the same line, there is exactly one plane.
   d. A line contains at least one point.
   e. If two lines are parallel, then their intersection is exactly one point.
   f. Through any two points, there is at most one line.

2. Determine whether each statement is always, sometimes, or never true. If the statement is not always true, explain why.
   a. If two planes intersect, their intersection is a line.
   b. The midpoint of a segment divides the segment into two congruent segments.
   c. There is exactly one plane that contains three collinear points.
   d. If two lines intersect, their intersection is one point.

3. Use the walls, floor, and ceiling of your classroom to describe a model for each of the following geometric situations.
   a. two planes that intersect in a line
   b. two planes that do not intersect
   c. three planes that intersect in a point

Remember What You Learned

4. A good way to remember a new mathematical term is to relate it to a word you already know. Explain how the idea of a mathematical theorem is related to the idea of a scientific theory.
Points, Lines, and Planes  In geometry, a postulate is a statement that is accepted as true. Postulates describe fundamental relationships in geometry.

Postulate: Through any two points, there is exactly one line.
Postulate: Through any three points not on the same line, there is exactly one plane.
Postulate: A line contains at least two points.
Postulate: A plane contains at least three points not on the same line.
Postulate: If two points lie in a plane, then the line containing those points lies in the plane.
Postulate: If two lines intersect, then their intersection is exactly one point.
Postulate: If two planes intersect, then their intersection is a line.

Example Determine whether each statement is always, sometimes, or never true.

a. There is exactly one plane that contains points A, B, and C.
   Sometimes; if A, B, and C are collinear, they are contained in many planes. If they are noncollinear, then they are contained in exactly one plane.

b. Points E and F are contained in exactly one line.
   Always; the first postulate states that there is exactly one line through any two points.

c. Two lines intersect in two distinct points M and N.
   Never; the intersection of two lines is one point.

Exercises Use postulates to determine whether each statement is always, sometimes, or never true.

1. A line contains exactly one point.
2. Noncollinear points R, S, and T are contained in exactly one plane.
3. Any two lines ℓ and m intersect.
4. If points G and H are contained in plane M, then \( \overline{GH} \) is perpendicular to plane M.
5. Planes \( R \) and \( S \) intersect in point T.
6. If points A, B, and C are noncollinear, then segments \( \overline{AB}, \overline{BC} \), and \( \overline{CA} \) are contained in exactly one plane.

In the figure, \( \overline{AC} \) and \( \overline{DE} \) are in plane Q and \( \overline{AC} \parallel \overline{DE} \). State the postulate that can be used to show each statement is true.

7. Exactly one plane contains points F, B, and E.
8. \( \overline{BE} \) lies in plane Q.
Paragraph Proofs  A statement that can be proved true is called a **theorem**. You can use undefined terms, definitions, postulates, and already-proved theorems to prove other statements true.

A logical argument that uses deductive reasoning to reach a valid conclusion is called a **proof**. In one type of proof, a **paragraph proof**, you write a paragraph to explain why a statement is true.

**Example**  In \( \triangle ABC \), \( \overline{BD} \) is an angle bisector. Write a paragraph proof to show that \( \angle ABD \equiv \angle CBD \).

By definition, an angle bisector divides an angle into two congruent angles. Since \( \overline{BD} \) is an angle bisector, \( \angle ABC \) is divided into two congruent angles. Thus, \( \angle ABD \equiv \angle CBD \).

**Exercises**

1. Given that \( \angle A \equiv \angle D \) and \( \angle D \equiv \angle E \), write a paragraph proof to show that \( \angle A \equiv \angle E \).

2. It is given that \( \overline{BC} \equiv \overline{EF} \), \( M \) is the midpoint of \( \overline{BC} \), and \( N \) is the midpoint of \( \overline{EF} \). Write a paragraph proof to show that \( BM = EN \).

3. Given that \( S \) is the midpoint of \( \overline{QP} \), \( T \) is the midpoint of \( \overline{PR} \), and \( P \) is the midpoint of \( \overline{ST} \), write a paragraph proof to show that \( QS = TR \).  

---

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Determine the number of line segments that can be drawn connecting each pair of points.

1.

2.

Determine whether the following statements are always, sometimes, or never true. Explain.

3. Three collinear points determine a plane.

4. Two points \( A \) and \( B \) determine a line.

5. A plane contains at least three lines.

In the figure, \( \overline{DG} \) and \( \overline{DP} \) lie in plane \( J \) and \( H \) lies on \( \overline{DG} \). State the postulate that can be used to show each statement is true.

6. \( G \) and \( P \) are collinear.

7. Points \( D \), \( H \), and \( P \) are coplanar.

8. **PROOF** In the figure at the right, point \( B \) is the midpoint of \( \overline{AC} \) and point \( C \) is the midpoint of \( \overline{BD} \). Write a paragraph proof to prove that \( AB = CD \).
Determine the number of line segments that can be drawn connecting each pair of points.

1. \[
\begin{array}{c}
\bullet \\
\bullet \\
\bullet
\end{array}
\]

2. \[
\begin{array}{c}
\bullet \\
\bullet \\
\bullet
\end{array}
\]

Determine whether the following statements are always, sometimes, or never true. Explain.

3. The intersection of two planes contains at least two points.

4. If three planes have a point in common, then they have a whole line in common.

In the figure, line \( m \) and \( \overline{TQ} \) lie in plane \( A \). State the postulate that can be used to show that each statement is true.

5. \( L, T, \) and line \( m \) lie in the same plane.

6. Line \( m \) and \( \overline{ST} \) intersect at \( T \).

7. In the figure, \( E \) is the midpoint of \( \overline{AB} \) and \( \overline{CD} \), and \( AB = CD \). Write a paragraph proof to prove that \( AE \cong ED \).

8. LOGIC Points \( A, B, \) and \( C \) are not collinear. Points \( B, C, \) and \( D \) are not collinear. Points \( A, B, C, \) and \( D \) are not coplanar. Describe two planes that intersect in line \( BC \).
2-5 Word Problem Practice

Postulates and Paragraph Proofs

1. ROOFING  Noel and Kirk are building a new roof. They wanted a roof with two sloping planes that meet along a curved arch. Is this possible?

2. AIRLINES  An airline company wants to provide service to San Francisco, Los Angeles, Chicago, Dallas, Washington D. C., and New York City. The company CEO draws lines between each pair of cities in the list on a map. No three of the cities are collinear. How many lines did the CEO draw?

3. TRIANGULATION  A sailor spots a whale through her binoculars. She wonders how far away the whale is, but the whale does not show up on the radar system. She sees another boat in the distance and radios the captain asking him to spot the whale and record its direction. Explain how this added information could enable the sailor to pinpoint the location of the whale. Under what circumstance would this idea fail?

4. POINTS  Carson claims that a line can intersect a plane at only one point and draws this picture to show his reasoning.

Zoe thinks it is possible for a line to intersect a plane at more than one point. Who is correct? Explain.

5. What is the maximum number of line segments that can be drawn between pairs among the 16 points?

6. When the owner finished the picture, he found that his company was split into two groups, one with 10 people and the other with 6. The people within a group were all friends, but nobody from one group was a friend of anybody from the other group. How many line segments were there?
Even and Odd

It is commonly known that to determine if a number is even, you check to see if the last number is divisible by 2. However, this is not the definition of an even number. The definition of an even number states that a number is even if it can be written as $2k$ for some integer $k$.

The following proof uses this definition to show that the sum of two even numbers is even.

Suppose $m$ and $n$ are even. By the definition, they can be written as $m = 2\ell$ and $n = 2j$ for some integers $\ell$ and $j$. We need to show that $m + n$ can be written as $2k$ for some integer $k$ to prove that the sum is even. Now, the sum $m + n$ can be written as $2\ell + 2j$ or $2(\ell + j)$ using the distributive property. Since $\ell$ and $j$ are both integers, the sum $\ell + j$ is equal to some integer $k$. So, $m + n$ can be written as $2k$ for some integer $k$. Therefore, the sum $m + n$ is even.

The definition of an odd number states that a number is odd if it can be written as $2k + 1$ for some integer $k$.

Use the definitions of even and odd numbers to write paragraph proof for each statement.

1. The sum of two odd numbers is even.

2. The product of two odd numbers is odd.

3. The product of two even numbers is even.
2-6 Lesson Reading Guide

Algebraic Proof

Get Ready for the Lesson

Read the introduction to Lesson 2-6 in your textbook.

What are some of the things that lawyers might use in presenting their closing arguments to a trial jury in addition to evidence gathered prior to the trial and testimony heard during the trial?

Read the Lesson

1. Name the property illustrated by each statement.
   a. If \( a = 4.75 \) and \( 4.75 = b \), then \( a = b \).
   b. If \( x = y \), then \( x + 8 = y + 8 \).
   c. \( 5(12 + 19) = 5 \cdot 12 + 5 \cdot 19 \)
   d. If \( x = 5 \), then \( x \) may be replaced with 5 in any equation or expression.
   e. If \( x = y \), then \( 8x = 8y \).
   f. If \( x = 23.45 \), then \( 23.45 = x \).
   g. If \( 5x = 7 \), then \( x = \frac{7}{5} \).
   h. If \( x = 12 \), then \( x - 3 = 9 \).

2. Give the reason for each statement in the following two-column proof.
   **Given:** \( 5(n - 3) = 4(2n - 7) - 14 \)
   **Prove:** \( n = 9 \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( 5(n - 3) = 4(2n - 7) - 14 )</td>
<td>1.</td>
</tr>
<tr>
<td>2. ( 5n - 15 = 8n - 28 - 14 )</td>
<td>2.</td>
</tr>
<tr>
<td>3. ( 5n - 15 = 8n - 42 )</td>
<td>3.</td>
</tr>
<tr>
<td>4. ( 5n - 15 + 15 = 8n - 42 + 15 )</td>
<td>4.</td>
</tr>
<tr>
<td>5. ( 5n = 8n - 27 )</td>
<td>5.</td>
</tr>
<tr>
<td>6. ( 5n - 8n = 8n - 27 - 8n )</td>
<td>6.</td>
</tr>
<tr>
<td>7. ( -3n = -27 )</td>
<td>7.</td>
</tr>
<tr>
<td>8. ( -\frac{3n}{-3} = -\frac{27}{-3} )</td>
<td>8.</td>
</tr>
<tr>
<td>9. ( n = 9 )</td>
<td>9.</td>
</tr>
</tbody>
</table>

Remember What You Learned

3. A good way to remember mathematical terms is to relate them to words you already know. Give an everyday word that is related in meaning to the mathematical term reflexive and explain how this word can help you to remember the Reflexive Property and to distinguish it from the Symmetric and Transitive Properties.
Algebraic Proof

The following properties of algebra can be used to justify the steps when solving an algebraic equation.

<table>
<thead>
<tr>
<th>Property</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive</td>
<td>For every number ( a ), ( a = a ).</td>
</tr>
<tr>
<td>Symmetric</td>
<td>For all numbers ( a ) and ( b ), if ( a = b ) then ( b = a ).</td>
</tr>
<tr>
<td>Transitive</td>
<td>For all numbers ( a ), ( b ), and ( c ), if ( a = b ) and ( b = c ) then ( a = c ).</td>
</tr>
<tr>
<td>Addition and Subtraction</td>
<td>For all numbers ( a ), ( b ), and ( c ), if ( a = b ) then ( a + c = b + c ) and ( a - c = b - c ).</td>
</tr>
<tr>
<td>Multiplication and Division</td>
<td>For all numbers ( a ), ( b ), and ( c ), if ( a = b ) then ( a \cdot c = b \cdot c ), and if ( c \neq 0 ) then ( \frac{a}{c} = \frac{b}{c} ).</td>
</tr>
<tr>
<td>Substitution</td>
<td>For all numbers ( a ) and ( b ), if ( a = b ) then ( a ) may be replaced by ( b ) in any equation or expression.</td>
</tr>
<tr>
<td>Distributive</td>
<td>For all numbers ( a ), ( b ), and ( c ), ( a(b + c) = ab + ac ).</td>
</tr>
</tbody>
</table>

Example

Solve \( 6x + 2(x - 1) = 30 \).

### Algebraic Steps

1. **Given**: \( \frac{4x + 6}{2} = 9 \)
2. **Prove**: \( x = 3 \)

#### Properties

- **Given**
- **Distributive Property**
- **Substitution**
- **Addition Property**
- **Substitution**
- **Division Property**
- **Substitution**

#### Solution

- \( 6x + 2(x - 1) = 30 \)
- \( 6x + 2x - 2 = 30 \)
- \( 8x - 2 = 30 \)
- \( 8x = 32 \)
- \( x = 4 \)

Exercises

Complete each proof.

1. **Given**: \( \frac{4x + 6}{2} = 9 \)
   **Prove**: \( x = 3 \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( \frac{4x + 6}{2} = 9 )</td>
<td>a. ________</td>
</tr>
<tr>
<td>b. ( \frac{4x + 6}{2} = 9 )</td>
<td>b. ________</td>
</tr>
<tr>
<td>c. ( 4x + 6 = 18 )</td>
<td>c. ________</td>
</tr>
<tr>
<td>d. ( 4x + 6 - 6 = 18 - 6 )</td>
<td>d. ________</td>
</tr>
<tr>
<td>e. ( 4x = 32 )</td>
<td>e. ________</td>
</tr>
<tr>
<td>f. ( \frac{4x}{4} = 8 )</td>
<td>f. ________</td>
</tr>
<tr>
<td>g. ( x = 4 )</td>
<td>g. ________</td>
</tr>
</tbody>
</table>

2. **Given**: \( 4x + 8 = x + 2 \)
   **Prove**: \( x = -2 \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( 4x + 8 = x + 2 )</td>
<td>a. ________</td>
</tr>
<tr>
<td>b. ( 4x + 8 - x = x + 2 - x )</td>
<td>b. ________</td>
</tr>
<tr>
<td>c. ( 3x + 8 = 2 )</td>
<td>c. ________</td>
</tr>
<tr>
<td>d. ( 3x = -6 )</td>
<td>d. ________</td>
</tr>
<tr>
<td>e. ( \frac{3x}{3} = \frac{-6}{3} )</td>
<td>e. ________</td>
</tr>
<tr>
<td>f. ________</td>
<td>f. ________</td>
</tr>
<tr>
<td>g. ________</td>
<td>g. ________</td>
</tr>
</tbody>
</table>
Geometric Proof  Geometry deals with numbers as measures, so geometric proofs use properties of numbers. Here are some of the algebraic properties used in proofs.

<table>
<thead>
<tr>
<th>Property</th>
<th>Segments</th>
<th>Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive</td>
<td>$AB = AB$</td>
<td>$m \angle A = m \angle A$</td>
</tr>
<tr>
<td>Symmetric</td>
<td>If $AB = CD$, then $CD = AB$.</td>
<td>If $m \angle A = m \angle B$, then $m \angle B = m \angle A$.</td>
</tr>
<tr>
<td>Transitive</td>
<td>If $AB = CD$ and $CD = EF$, then $AB = EF$.</td>
<td>If $m \angle 1 = m \angle 2$ and $m \angle 2 = m \angle 3$, then $m \angle 1 = m \angle 3$.</td>
</tr>
</tbody>
</table>

**Example**  Write a two-column proof.

**Given:** $m \angle 1 = m \angle 2$, $m \angle 2 = m \angle 3$

**Prove:** $m \angle 1 = m \angle 3$

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $m \angle 1 = m \angle 2$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $m \angle 2 = m \angle 3$</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. $m \angle 1 = m \angle 3$</td>
<td>3. Transitive Property</td>
</tr>
</tbody>
</table>

**Exercises**

State the property that justifies each statement.

1. If $m \angle 1 = m \angle 2$, then $m \angle 2 = m \angle 1$.
2. If $m \angle 1 = 90$ and $m \angle 2 = m \angle 1$, then $m \angle 2 = 90$.
3. If $AB = RS$ and $RS = WY$, then $AB = WY$.
4. If $AB = CD$, then $\frac{1}{2}AB = \frac{1}{2}CD$.
5. If $m \angle 1 + m \angle 2 = 110$ and $m \angle 2 = m \angle 3$, then $m \angle 1 + m \angle 3 = 110$.
6. $RS = RS$
7. If $AB = RS$ and $TU = WY$, then $AB + TU = RS + WY$.
8. If $m \angle 1 = m \angle 2$ and $m \angle 2 = m \angle 3$, then $m \angle 1 = m \angle 3$.

9. A formula for the area of a triangle is $A = \frac{1}{2}bh$. Prove that $bh$ is equal to 2 times the area of the triangle.
Skills Practice
Algebraic Proof

State the property that justifies each statement.

1. If \( 80 = m\angle A \), then \( m\angle A = 80 \).

2. If \( RS = TU \) and \( TU = YP \), then \( RS = YP \).

3. If \( 7x = 28 \), then \( x = 4 \).

4. If \( VR + TY = EN + TY \), then \( VR = EN \).

5. If \( m\angle 1 = 30 \) and \( m\angle 1 = m\angle 2 \), then \( m\angle 2 = 30 \).

Complete the following proof.

6. Given: \( 8x - 5 = 2x + 1 \)
Prove: \( x = 1 \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( 8x - 5 = 2x + 1 )</td>
<td>a.</td>
</tr>
<tr>
<td>b. ( 8x - 5 - 2x = 2x + 1 - 2x )</td>
<td>b.</td>
</tr>
<tr>
<td>c.</td>
<td>c. Substitution Property</td>
</tr>
<tr>
<td>d.</td>
<td>d. Addition Property</td>
</tr>
<tr>
<td>e. ( 6x = 6 )</td>
<td>e.</td>
</tr>
<tr>
<td>f. ( \frac{6x}{6} = \frac{6}{6} )</td>
<td>f.</td>
</tr>
<tr>
<td>g.</td>
<td>g.</td>
</tr>
</tbody>
</table>

Write a two-column proof for the following.

7. If \( PQ \equiv QS \) and \( QS \equiv ST \), then \( PQ = ST \).
2-6 \hspace{1cm} \textbf{Practice}

\textit{Algebraic Proof}

\textbf{PROOF} Write a two-column proof.

1. If $m\angle ABC + m\angle CBD = 90$, $m\angle ABC = 3x - 5$, and $m\angle CBD = \frac{x + 1}{2}$, then $x = 27$.

2. \textbf{FINANCE} The formula for simple interest is $I = prt$, where $I$ is interest, $p$ is principal, $r$ is rate, and $t$ is time. Solve the formula for $r$ and justify each step.
2-6 Word Problem Practice

Algebraic Proof

1. **DOGS** Jessica and Robert each own the same number of dogs. Robert and Gail also own the same number of dogs. Without knowing how many dogs they own, one can still conclude that Jessica and Gail each own the same number of dogs. What property is used to make this conclusion?

2. **MONEY** Lars and Peter both have the same amount of money in their wallets. They went to the store together and decided to buy some cookies, splitting the cost equally. After buying the cookies, do they still have the same amount of money in their wallets? What property is relevant to help you decide?

4. **MANUFACTURING** A company manufactures small electronic components called diodes. Each diode is worth $1.50. Plant A produced 4,443 diodes and Plant B produced 5,557 diodes. The foreman was asked what the total value of all the diodes was. The foreman immediately responded “$15,000.” The foreman would not have been able to compute the value so quickly if he had to multiply $1.50 by 4,443 and then add this to the result of $1.50 times 5,557. Explain how you think the foreman got the answer so quickly?

4. **FIGURINES** Pete and Rhonda paint figurines. They can both paint 8 figurines per hour. One day, Pete worked 6 hours while Rhonda worked 9 hours. How many figurines did they paint that day? Show how to get the answer using the Distributive Property.

AGE For Exercises 5 and 6, use the following information.

William’s father is eight years older than 4 times William’s age. William’s father is 36 years old.

5. Let \( x \) be William’s age. Translate the given information into an algebraic equation involving \( x \).

6. Fill in the missing steps and justifications for each step in finding the value of \( x \).

<table>
<thead>
<tr>
<th>Algebraic Steps</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 4x + 8 = 36 )</td>
<td>Original equation</td>
</tr>
<tr>
<td>( 4x = 28 )</td>
<td>Subtraction Property</td>
</tr>
<tr>
<td>( \frac{4x}{4} = \frac{28}{4} )</td>
<td>Substitution Property</td>
</tr>
</tbody>
</table>
Symmetric, Reflexive, and Transitive Properties

Equality has three important properties.

- Reflexive: \( a = a \)
- Symmetric: If \( a = b \), then \( b = a \).
- Transitive: If \( a = b \) and \( b = c \), then \( a = c \).

Other relations have some of the same properties. Consider the relation “is next to” for objects labeled \( X \), \( Y \), and \( Z \). Which of the properties listed above are true for this relation?

- \( X \) is next to \( X \). False
- If \( X \) is next to \( Y \), then \( Y \) is next to \( X \). True
- If \( X \) is next to \( Y \) and \( Y \) is next to \( Z \), then \( X \) is next to \( Z \). False

Only the symmetric property is true for the relation “is next to.”

For each relation, state which properties (symmetric, reflexive, transitive) are true.

1. is the same size as
2. is a family descendant of
3. is in the same room as
4. is the identical twin of
5. is warmer than
6. is on the same line as
7. is a sister of
8. is the same weight as
9. Find two other examples of relations, and tell which properties are true for each relation.
Lesson 2-7
Proving Segment Relationships

Get Ready for the Lesson

Read the introduction to Lesson 2-7 in your textbook.

- What is the total distance that the plane will fly to get from San Diego to Dallas?

- Before leaving home, a passenger used a road atlas to determine that the distance between San Diego and Dallas is about 1350 miles. Why is the flying distance greater than that?

Read the Lesson

1. If $E$ is between $Y$ and $S$, which of the following statements are always true?

   A. $YS + ES = YE$
   B. $YS - ES = YE$
   C. $YE > ES$
   D. $YE \cdot ES = YS$
   E. $SE + EY = SY$
   F. $E$ is the midpoint of $YS$.

2. Give the reason for each statement in the following two-column proof.

   **Given:** $C$ is the midpoint of $BD$.
   $D$ is the midpoint of $CE$.

   **Prove:** $BD \cong CE$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $C$ is the midpoint of $BD$.</td>
<td>1. $C$ is the midpoint of $BD$.</td>
</tr>
<tr>
<td>2. $BC = CD$</td>
<td>2. $BC = CD$</td>
</tr>
<tr>
<td>3. $D$ is the midpoint of $CE$.</td>
<td>3. $D$ is the midpoint of $CE$.</td>
</tr>
<tr>
<td>4. $CD = DE$</td>
<td>4. $CD = DE$</td>
</tr>
<tr>
<td>5. $BC = DE$</td>
<td>5. $BC = DE$</td>
</tr>
<tr>
<td>6. $BC + CD = CD + DE$</td>
<td>6. $BC + CD = CD + DE$</td>
</tr>
<tr>
<td>7. $BC + CD = BD$</td>
<td>7. $BC + CD = BD$</td>
</tr>
<tr>
<td>$CD + DE = CE$</td>
<td>$CD + DE = CE$</td>
</tr>
<tr>
<td>8. $BD = CE$</td>
<td>8. $BD = CE$</td>
</tr>
<tr>
<td>9. $BD \cong CE$</td>
<td>9. $BD \cong CE$</td>
</tr>
</tbody>
</table>

Remember What You Learned

3. One way to keep the names of related postulates straight in your mind is to associate something in the name of the postulate with the content of the postulate. How can you use this idea to distinguish between the Ruler Postulate and the Segment Addition Postulate?
Segment Addition  Two basic postulates for working with segments and lengths are the Ruler Postulate, which establishes number lines, and the Segment Addition Postulate, which describes what it means for one point to be between two other points.

Ruler Postulate  The points on any line or line segment can be paired with real numbers so that, given any two points A and B on a line, A corresponds to zero and B corresponds to a positive real number.

Segment Addition Postulate  B is between A and C if and only if \( AB + BC = AC \).

**Example**  Write a two-column proof.

*Given:* Q is the midpoint of PR.  
R is the midpoint of QS.

*Prove:* PR = QS

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Q is the midpoint of PR.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. PQ = QR</td>
<td>2. Definition of midpoint</td>
</tr>
<tr>
<td>3. R is the midpoint of QS.</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. QR = RS</td>
<td>4. Definition of midpoint</td>
</tr>
<tr>
<td>5. PQ + QR = QR + RS</td>
<td>5. Addition Property</td>
</tr>
<tr>
<td>7. PQ + QR = PR, QR + RS = QS</td>
<td>7. Segment Addition Postulate</td>
</tr>
<tr>
<td>8. PR = QS</td>
<td>8. Substitution</td>
</tr>
</tbody>
</table>

**Exercises**  Complete each proof.

1. **Given:** BC = DE  
   **Prove:** AB + DE = AC

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. BC = DE</td>
<td>a.</td>
</tr>
<tr>
<td>c. AB + DE = AC</td>
<td>c.</td>
</tr>
</tbody>
</table>

2. **Given:** Q is between P and R, R is between Q and S, PR = QS.  
   **Prove:** PQ = RS

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Q is between P and R.</td>
<td>a. Given</td>
</tr>
<tr>
<td>b. PQ + QR = PR</td>
<td>b.</td>
</tr>
<tr>
<td>c. R is between Q and S.</td>
<td>c.</td>
</tr>
<tr>
<td>e. PR = QS</td>
<td>e.</td>
</tr>
<tr>
<td>f. PQ + QR = QR + RS</td>
<td>f.</td>
</tr>
<tr>
<td>g. PQ + QR - QR = QR + RS - QR</td>
<td>g.</td>
</tr>
<tr>
<td>h.</td>
<td>h. Substitution</td>
</tr>
</tbody>
</table>
Study Guide and Intervention

Proving Segment Relationships

Segment Congruence

Three properties of algebra—the Reflexive, Symmetric, and Transitive Properties of Equality—have counterparts as properties of geometry. These properties can be proved as a theorem. As with other theorems, the properties can then be used to prove relationships among segments.

<table>
<thead>
<tr>
<th>Segment Congruence Theorem</th>
<th>Congruence of segments is reflexive, symmetric, and transitive.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive Property</td>
<td>( \overline{AB} \equiv \overline{AB} )</td>
</tr>
<tr>
<td>Symmetric Property</td>
<td>If ( \overline{AB} \equiv \overline{CD} ), then ( \overline{CD} \equiv \overline{AB} ).</td>
</tr>
<tr>
<td>Transitive Property</td>
<td>If ( \overline{AB} \equiv \overline{CD} ) and ( \overline{CD} \equiv \overline{EF} ), then ( \overline{AB} \equiv \overline{EF} ).</td>
</tr>
</tbody>
</table>

**Example**

Write a two-column proof.

Given: \( \overline{AB} \equiv \overline{DE} \); \( \overline{BC} \equiv \overline{EF} \)

Prove: \( \overline{AC} \equiv \overline{DF} \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{AB} \equiv \overline{DE} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \overline{BC} \equiv \overline{EF} )</td>
<td>2. Definition of congruence of segments</td>
</tr>
<tr>
<td>3. ( \overline{AC} \equiv \overline{DF} )</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( \overline{BC} = \overline{EF} )</td>
<td>4. Definition of congruence of segments</td>
</tr>
<tr>
<td>5. ( \overline{AB} + \overline{BC} = \overline{DE} + \overline{EF} )</td>
<td>5. Addition Property</td>
</tr>
<tr>
<td>6. ( \overline{AC} = \overline{DE} + \overline{EF} = \overline{DF} )</td>
<td>6. Segment Addition Postulate</td>
</tr>
<tr>
<td>7. ( \overline{AC} = \overline{DF} )</td>
<td>7. Substitution</td>
</tr>
<tr>
<td>8. ( \overline{AC} \equiv \overline{DF} )</td>
<td>8. Definition of congruence of segments</td>
</tr>
</tbody>
</table>

**Exercises**

Justify each statement with a property of congruence.

1. If \( \overline{DE} \equiv \overline{GH} \), then \( \overline{GH} \equiv \overline{DE} \).

2. If \( \overline{AB} \equiv \overline{RS} \) and \( \overline{RS} \equiv \overline{WY} \), then \( \overline{AB} \equiv \overline{WY} \).

3. \( \overline{RS} \equiv \overline{RS} \)

4. Complete the proof.
   
   Given: \( \overline{PR} \equiv \overline{QS} \)
   
   Prove: \( \overline{PQ} \equiv \overline{RS} \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( \overline{PR} \equiv \overline{QS} )</td>
<td>a. ____________________________</td>
</tr>
<tr>
<td>b. ( \overline{PR} = \overline{QS} )</td>
<td>b. ____________________________</td>
</tr>
<tr>
<td>c. ( \overline{PQ} + \overline{QR} = \overline{PR} )</td>
<td>c. ____________________________</td>
</tr>
<tr>
<td>d. ( \overline{PQ} + \overline{QR} = \overline{QR} + \overline{RS} )</td>
<td>d. Segment Addition Postulate</td>
</tr>
<tr>
<td>e. ( \overline{QR} + \overline{RS} )</td>
<td>e. ____________________________</td>
</tr>
<tr>
<td>f. Subtraction Property</td>
<td>f. ____________________________</td>
</tr>
<tr>
<td>g. Definition of congruence of segments</td>
<td>g. ____________________________</td>
</tr>
</tbody>
</table>
Skills Practice

Proving Segment Relationships

Justify each statement with a property of equality, a property of congruence, or a postulate.

1. \( QA = QA \)

2. If \( AB \cong BC \) and \( BC \cong CE \), then \( AB \cong CE \).

3. If \( Q \) is between \( P \) and \( R \), then \( PR = PQ + QR \).

4. If \( AB + BC = EF + FG \) and \( AB + BC = AC \), then \( EF + FG = AC \).

Complete each proof.

5. Given: \( SU \cong LR \)
   \( TU \cong LN \)
   Prove: \( ST \cong NR \)

   Proof:
<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( SU \cong LR, TU \cong LN )</td>
<td>a.</td>
</tr>
<tr>
<td>b.</td>
<td>b. Definition of ( \cong ) segments</td>
</tr>
<tr>
<td>c. ( SU = ST + TU ) ( LR = LN + NR )</td>
<td>c.</td>
</tr>
<tr>
<td>d. ( ST + TU = LN + NR )</td>
<td>d.</td>
</tr>
<tr>
<td>e. ( ST + LN = LN + NR )</td>
<td>e.</td>
</tr>
<tr>
<td>f. ( ST + LN - LN = LN + NR - LN )</td>
<td>f.</td>
</tr>
<tr>
<td>g. ( ST \cong NR )</td>
<td>g. Substitution Property</td>
</tr>
<tr>
<td>h.</td>
<td>h.</td>
</tr>
</tbody>
</table>

6. Given: \( AB \cong CD \)
   Prove: \( CD \cong AB \)

   Proof:
<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>a. Given</td>
</tr>
<tr>
<td>b. ( AB = CD )</td>
<td>b.</td>
</tr>
<tr>
<td>c. ( CD = AB )</td>
<td>c.</td>
</tr>
<tr>
<td>d.</td>
<td>d. Definition of ( \cong ) segments</td>
</tr>
</tbody>
</table>
Practice

2-7

Proving Segment Relationships

Complete the following proof.

1. Given: \( \overline{AB} \equiv \overline{DE} \)
   \( B \) is the midpoint of \( \overline{AC} \).
   \( E \) is the midpoint of \( \overline{DF} \).

Prove: \( \overline{BC} \equiv \overline{EF} \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>a. Given</td>
</tr>
<tr>
<td>b.</td>
<td>b. ( AB = DE )</td>
</tr>
<tr>
<td>c.</td>
<td>c. Definition of Midpoint</td>
</tr>
<tr>
<td>d. ( BC = DE )</td>
<td>d. ( BC = EF )</td>
</tr>
<tr>
<td>e. ( BC = EF )</td>
<td>e. ( BC = EF )</td>
</tr>
<tr>
<td>f.</td>
<td>f. ( BC = EF )</td>
</tr>
</tbody>
</table>

2. TRAVEL Refer to the figure. DeAnne knows that the distance from Grayson to Apex is the same as the distance from Redding to Pine Bluff. Prove that the distance from Grayson to Redding is equal to the distance from Apex to Pine Bluff.
1. FAMILY  Maria is 11 inches shorter than her sister Nancy. Brad is 11 inches shorter than his brother Chad. If Maria is shorter than Brad, how do the heights of Nancy and Chad compare? What if Maria and Brad are the same height?

4. NEIGHBORHOODS  Karla, John, and Mandy live in three houses that are on the same line. John lives between Karla and Mandy. Karla and Mandy live a mile apart. Is it possible for John to be a mile from both Karla and Mandy?

2. DISTANCE  Martha and Laura live 1,400 meters apart. A library is opened between them and is 500 meters from Martha.

   How far is the library from Laura?

3. LUMBER  Byron works in a lumber yard. His boss just cut a dozen planks and asked Byron to double check that they are all the same length. The planks were numbered 1 through 12. Byron took out plank number 1 and checked that the other planks are all the same length as plank 1. He concluded that they must all be the same length. Explain how you know plank 7 and plank 10 are the same length even though they were never directly compared to each other?

5. LIGHTS  For Exercises 5 and 6, use the following information.

   Five lights, A, B, C, D, and E, are lined up in a row. The middle light is the midpoint of the second and fourth light and also the midpoint of the first and last light.

   5. Draw a figure to illustrate the situation.
   6. Complete this proof.

   Given:  C is the midpoint of $BD$ and $AE$.
   Prove:  $AB = DE$

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $C$ is the midpoint of $BD$ and $AE$.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $BC = CD$ and</td>
<td>2.</td>
</tr>
<tr>
<td>3. $AC = AB + BC$, $CE = CD + DE$</td>
<td>3.</td>
</tr>
<tr>
<td>4. $AB = AC - BC$</td>
<td>4.</td>
</tr>
<tr>
<td>5.</td>
<td>5. Substitution Property</td>
</tr>
<tr>
<td>7.</td>
<td>7.</td>
</tr>
</tbody>
</table>
Midpoint Counterpoint

The midpoint $M$ of $\overline{AB}$ when $A$ is $(x_1, y_1)$ and $B$ is $(x_2, y_2)$ is found by using the formula $M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.

Suppose point $P$ is a point on $\overline{AB}$ located $\frac{1}{4}$ of the distance from $A$ to $B$. Juan says the coordinates of $P$ can be found by using the formula $P = \left( \frac{x_1 + x_2}{4}, \frac{y_1 + y_2}{4} \right)$.

1. Is Juan’s formula for $P$ valid? Explain your answer.

2. Use midpoints to find a formula for the coordinates of $P$. Write your formula in terms of $x_1, y_1, x_2, y_2$.

For Exercises 3–5, use the coordinate plane at the right.

3. Graph $A(2, -2)$ and $B(14, 4)$.

4. Graph point $P$ between $A$ and $B$ so that $AP$ is $\frac{1}{4}(AB)$. What are its coordinates?

5. Graph point $C$ so that $B$ is between $A$ and $C$ and $BC$ is $\frac{1}{4}(AB)$. What are the coordinates of point $C$?
Lesson Reading Guide

Proving Angle Relationships

Get Ready for the Lesson

Read the introduction to Lesson 2-8 in your textbook.

Is it possible to open a pair of scissors so that the angles formed by the two blades, a blade and a handle, and the two handles, are all congruent? If so, explain how this could happen.

Read the Lesson

1. Complete each sentence to form a statement that is always true.
   a. If two angles form a linear pair, then they are adjacent and _________.
   b. If two angles are complementary to the same angle, then they are _________.
   c. If \( D \) is a point in the interior of \( \angle ABC \), then \( m\angle ABC = m\angle ABD + \) _________.
   d. Given \( \overline{RS} \) and a number \( x \) between ______ and _______, there is exactly one ray with endpoint \( R \), extended on either side of \( RS \), such that the measure of the angle formed is \( x \).
   e. If two angles are congruent and supplementary, then each angle is a(n) _________.
   f. ________ lines form congruent adjacent angles.
   g. “Every angle is congruent to itself” is a statement of the __________ Property of angle congruence.
   h. If two congruent angles form a linear pair, then the measure of each angle is _______.
   i. If the noncommon sides of two adjacent angles form a right angle, then the angles are _________.

2. Determine whether each statement is always, sometimes, or never true.
   a. Supplementary angles are congruent.
   b. If two angles form a linear pair, they are complementary.
   c. Two vertical angles are supplementary.
   d. Two adjacent angles form a linear pair.
   e. Two vertical angles form a linear pair.
   f. Complementary angles are congruent.
   g. Two angles that are congruent to the same angle are congruent to each other.
   h. Complementary angles are adjacent angles.

Remember What You Learned

3. A good way to remember something is to explain it to someone else. Suppose that a classmate thinks that two angles can only be vertical angles if one angle lies above the other. How can you explain to him the meaning of vertical angles, using the word vertex in your explanation?
2-8 Study Guide and Intervention

Proving Angle Relationships

Supplementary and Complementary Angles There are two basic postulates for working with angles. The Protractor Postulate assigns numbers to angle measures, and the Angle Addition Postulate relates parts of an angle to the whole angle.

<table>
<thead>
<tr>
<th>Protractor Postulate</th>
<th>Given $\overline{AB}$ and a number $r$ between 0 and 180, there is exactly one ray with endpoint $A$, extending on either side of $\overline{AB}$, such that the measure of the angle formed is $r$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle Addition Postulate</td>
<td>$R$ is in the interior of $\angle PQS$ if and only if $m\angle PQR + m\angle RQS = m\angle PQS$.</td>
</tr>
</tbody>
</table>

The two postulates can be used to prove the following two theorems.

<table>
<thead>
<tr>
<th>Supplement Theorem</th>
<th>If two angles form a linear pair, then they are supplementary angles. If $\angle 1$ and $\angle 2$ form a linear pair, then $m\angle 1 + m\angle 2 = 180$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complement Theorem</td>
<td>If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles. If $\overline{GF} \perp \overline{GH}$, then $m\angle 3 + m\angle 4 = 90$.</td>
</tr>
</tbody>
</table>

**Example 1** If $\angle 1$ and $\angle 2$ form a linear pair and $m\angle 2 = 115$, find $m\angle 1$.

$m\angle 1 + m\angle 2 = 180$ Suppl. Theorem
$m\angle 1 + 115 = 180$ Substitution
$m\angle 1 = 65$ Subtraction Prop.

**Example 2** If $\angle 1$ and $\angle 2$ form a right angle and $m\angle 2 = 20$, find $m\angle 1$.

$m\angle 1 + m\angle 2 = 90$ Compl. Theorem
$m\angle 1 + 20 = 90$ Substitution
$m\angle 1 = 70$ Subtraction Prop.

**Exercises**

Find the measure of each numbered angle.

1. $m\angle 7 = 5x + 5$,
   $m\angle 8 = x - 5$
2. $m\angle 5 = 5x$, $m\angle 6 = 4x + 6$,
   $m\angle 7 = 10x$,
   $m\angle 8 = 12x - 12$
3. $m\angle 11 = 11x$,
   $m\angle 12 = 10x + 10$
2-8

Study Guide and Intervention (continued)

Proving Angle Relationships

Congruent and Right Angles

Three properties of angles can be proved as theorems.

<table>
<thead>
<tr>
<th>Congruence of angles is reflexive, symmetric, and transitive.</th>
<th>Angles complementary to the same angle or to congruent angles are congruent.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td><img src="image" alt="Diagram" /></td>
</tr>
</tbody>
</table>

If $\angle 1$ and $\angle 2$ are supplementary to $\angle 3$, then $\angle 1 \equiv \angle 2$.

If $\angle 4$ and $\angle 5$ are complementary to $\angle 6$, then $\angle 4 \equiv \angle 5$.

Example

Write a two-column proof.

Given: $\angle ABC$ and $\angle CBD$ are complementary. $\triangle DBE$ and $\angle CBD$ form a right angle.

Prove: $\angle ABC \equiv \angle DBE$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle ABC$ and $\angle CBD$ are complementary. $\triangle DBE$ and $\angle CBD$ form a right angle.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle DBE$ and $\angle CBD$ are complementary.</td>
<td>2. Complement Theorem</td>
</tr>
<tr>
<td>3. $\angle ABC \equiv \angle DBE$</td>
<td>3. Angles complementary to the same $\angle$ are $\equiv$.</td>
</tr>
</tbody>
</table>

Exercises

Complete each proof.

1. Given: $\overline{AB} \perp \overline{BC}$; $\angle 1$ and $\angle 3$ are complementary.

Prove: $\angle 2 \equiv \angle 3$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $\overline{AB} \perp \overline{BC}$</td>
<td>a. $\angle 1$ and $\angle 2$ form a linear pair. $m\angle 1 + m\angle 2 = 180$</td>
</tr>
<tr>
<td>b.</td>
<td>b. Definition of $\perp$</td>
</tr>
<tr>
<td>c. $m\angle ABC = 90$</td>
<td>c. Def. of right angle</td>
</tr>
<tr>
<td>d. $m\angle ABC = m\angle 1 + m\angle 2$</td>
<td>d. $m\angle 1 + m\angle 3 = 180$</td>
</tr>
<tr>
<td>e. $90 = m\angle 1 + m\angle 2$</td>
<td>e. Substitution</td>
</tr>
<tr>
<td>f. $\angle 1$ and $\angle 2$ are compl.</td>
<td>f. $\angle 1$ is suppl. to $\angle 3$.</td>
</tr>
<tr>
<td>g.</td>
<td>g. Given</td>
</tr>
<tr>
<td>h. $\angle 2 \equiv \angle 3$</td>
<td>h. $\triangle$ suppl. to the same $\angle$ are $\equiv$.</td>
</tr>
</tbody>
</table>

2. Given: $\angle 1$ and $\angle 2$ form a linear pair. $m\angle 1 + m\angle 3 = 180$

Prove: $\angle 2 \equiv \angle 3$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $\angle 1$ and $\angle 2$ form a linear pair.</td>
<td>a. Given</td>
</tr>
<tr>
<td>b. $m\angle 1 + m\angle 3 = 180$</td>
<td>b. Suppl. Theorem</td>
</tr>
<tr>
<td>c. $\angle 1$ is suppl. to $\angle 3$.</td>
<td>c. $\equiv$</td>
</tr>
<tr>
<td>d. $\triangle$ suppl. to the same $\angle$ are $\equiv$.</td>
<td>d. $\triangle$ suppl. to the same $\angle$ are $\equiv$.</td>
</tr>
</tbody>
</table>
2-8 Skills Practice

Proving Angle Relationships

Find the measure of each numbered angle.

1. \( m \angle 2 = 57 \) 
2. \( m \angle 5 = 22 \) 
3. \( m \angle 1 = 38 \)

\[
\begin{array}{cc}
1 & 2 \\
\end{array}
\]

\[
\begin{array}{cc}
5 & 6 \\
\end{array}
\]

4. \( m \angle 13 = 4x + 11, \quad m \angle 14 = 3x + 1 \)

5. \( \angle 9 \) and \( \angle 10 \) are complementary. 
\( \angle 7 \equiv \angle 9, \quad m \angle 8 = 41 \)

\[
\begin{array}{cc}
13 & 14 \\
\end{array}
\]

\[
\begin{array}{cc}
7 & 8 \quad 9 \quad 10 \\
\end{array}
\]

6. \( m \angle 2 = 4x - 26, \quad m \angle 3 = 3x + 4 \)

\[
\begin{array}{cc}
2 & 3 \\
\end{array}
\]

Determine whether the following statements are always, sometimes, or never true.

7. Two angles that are supplementary form a linear pair.

8. Two angles that are vertical are adjacent.

9. Copy and complete the following proof.

Given: \( \angle QPS \equiv \angle TPR \)

Prove: \( \angle QPR \equiv \angle TPS \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( \angle QPS ) \equiv ( \angle TPR )</td>
<td>a.</td>
</tr>
<tr>
<td>b. ( m \angle QPS = m \angle TPR )</td>
<td>b.</td>
</tr>
<tr>
<td>c. ( m \angle QPS = m \angle QPR + m \angle RPS ) \quad ( m \angle TPR = m \angle TPS + m \angle RPS )</td>
<td>c.</td>
</tr>
<tr>
<td>d. ( )</td>
<td>d. Substitution</td>
</tr>
<tr>
<td>e. ( )</td>
<td>e.</td>
</tr>
<tr>
<td>f. ( )</td>
<td>f.</td>
</tr>
</tbody>
</table>
Find the measure of each numbered angle.

1. \( m\angle 1 = x + 10 \)  
   \( m\angle 2 = 3x + 18 \)

2. \( m\angle 4 = 2x - 5 \)  
   \( m\angle 5 = 4x - 13 \)

3. \( m\angle 6 = 7x - 24 \)  
   \( m\angle 7 = 5x + 14 \)

Determine whether the following statements are \textit{always}, \textit{sometimes}, or \textit{never} true.

4. Two angles that are supplementary are complementary.

5. Complementary angles are congruent.

6. Write a two-column proof.
   \textbf{Given:} \( \angle 1 \) and \( \angle 2 \) form a linear pair.  
   \( \angle 2 \) and \( \angle 3 \) are supplementary.  
   \textbf{Prove:} \( \angle 1 \cong \angle 3 \)

7. \textbf{STREETS} Refer to the figure. Barton Road and Olive Tree Lane form a right angle at their intersection. Tryon Street forms a 57° angle with Olive Tree Lane. What is the measure of the acute angle Tryon Street forms with Barton Road?
Word Problem Practice

Proving Angle Relationships

1. **ICOSAHEDRA** For a school project, students are making a giant icosahedron, which is a large solid with many identical triangular faces. John is assigned quality control. He must make sure that the measures of all the angles in all the triangles are the same as each other. He does this by using a precut template and comparing the corner angles of every triangle to the template. How does this assure that the angles in all the triangles will be congruent to each other?

2. **VISTAS** If you look straight ahead at a scenic point, you can see a waterfall. If you turn your head 25° to the left, you will see a famous mountain peak. If you turn your head 35° more to the left, you will see another waterfall. If you are looking straight ahead, through how many degrees must you turn your head to the left in order to see the second waterfall?

3. **TUBES** A tube with a hexagonal cross section is placed on the floor.

   What is the measure of \( \angle 1 \) in the figure given that the angle at one corner of the hexagon is 120°?

4. **PAINTING** Students are painting their rectangular classroom ceiling. They want to paint a line that intersects one of the corners as shown in the figure.

   They want the painted line to make a 15° angle with one edge of the ceiling. Unfortunately, between the line and the edge there is a water pipe making it difficult to measure the angle. They decide to measure the angle to the other edge. Given that the corner is a right angle, what is the measure of the other angle?

For Exercises 5–7, use the following information.

Clyde looks at a building from point \( E \). \( \angle AEC \) has the same measure as \( \angle BED \).

5. The measure of \( \angle AEC \) is equal to the sum of the measures of \( \angle AEB \) and what other angle?

6. The measure of \( \angle BED \) is equal to the sum of the measures of \( \angle CED \) and what other angle?

7. Is it true that \( m \angle AEB \) is equal to \( m \angle CED \)?
2-8 Enrichment

Stars

There are many different types of stars. Stars can have 5 points, 6 points, 7 points, or more. The sum of the angles of the star changes depending on the number of points.

1. Find the sum of the measures of the angles in the 5-pointed star.

2. Find the sum of the measures of the angles in the 6-pointed star.

3. Complete the table for the sum of the measures of the angles in a star with the number of points given.

<table>
<thead>
<tr>
<th>Number of points</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of angles</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Make a conjecture about the formula for the sum of the measures of the angles for a star with \( n \) points. Using this formula, what will be the sum of the angles in a star with 12 points?

5. Use the figures at the right to determine if this formula will always work? Explain.