**Answers (Lesson 1-1)**

**Expressions and Formulas**

**Get Ready for the Lesson**

Read the introduction to Lesson 1-1 in your textbook.

- Nurses use the formula $F = \frac{D\times t}{V}$ to control the flow rate for IVs. Name the quantities that each of the variables in this formula represents and the units in which each is measured.

- Write a statement that a nurse would use to calculate the flow rate of an IV if a doctor orders 1350 milliliters of IV saline to be given over 8 hours, with a drop factor of 20 drops per milliliter. Do not include the actual calculation.

- Write an expression that a nurse would use to calculate the flow rate of an IV if a doctor orders 1350 milliliters of IV saline to be given over 8 hours, with a drop factor of 20 drops per milliliter, but uses a drop factor of 25 instead.

**Read the Lesson**

1. There are a customary order for grouping symbols. Brackets are used outside of parentheses. Parentheses are used inside of brackets. Identify the innermost expression(s) in each of the following expressions.
   - a. $(3 - 25) + (8 - 6) + (10 - 7)$
   - b. $14 - [8 - (13 - 9) + 21]$ + $10 - (8 - 100) (10 - 12)$

2. Write the expression that a nurse would use to calculate the flow rate of an IV if a doctor orders 1350 milliliters of IV saline to be given over 8 hours, with a drop factor of 20 drops per milliliter. Do not include the actual calculation.

**Remember What You learned**

1. Algebraic expressions contain at least one variable.
2. Algebraic expressions can be put in the form of a mathematical equation.

**Anticipation Guide**

1. What is the importance of a nurse using the formula $F = \frac{D\times t}{V}$ to control the flow rate for IVs?
2. How is the formula $F = \frac{D\times t}{V}$ used by a nurse to control the flow rate for IVs?
3. What is the importance of nurses using the formula $F = \frac{D\times t}{V}$ to control the flow rate for IVs?
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12. What is the importance of nurses using the formula $F = \frac{D\times t}{V}$ to control the flow rate for IVs?
Chapter 1

Expressions and Formulas

Example 1

Evaluate 3\(^2\) and 2 \times 4.

\[ 3^2 = 9 \quad \text{and} \quad 2 \times 4 = 8 \]

Example 2

Evaluate each expression.

(a) \( 14 + 6 = 20 \)
(b) \( 18 + 12 = 30 \)
(c) \( 21 + (3 + 2) = 26 \)
(d) \( 15 \times 2 = 30 \)
(e) \( 6 \div 2 = 3 \)

Exercise 1

Find the value of each expression.

1. \( 1.1 \times (3 + 2) = 10.1 \)
2. \( 12 \times 2 = 24 \)
3. \( 6 \div 2 = 3 \)
4. \( 15 \div 3 = 5 \)
5. \( 10 - 2 \times 3 = 4 \)
6. \( 8 \div 2 = 4 \)

Exercise 2

Evaluate each expression if \( x = 3, y = 2, z = 4 \), and \( t = 5 \).

1. \( x + y = 5 \)
2. \( y - t = -3 \)
3. \( z \times y = 16 \)
4. \( t \div z = 0.625 \)

Exercise 3

Simplify the expressions inside grouping symbols.

1. \( (2 + 3) \times (4 + 5) = 21 \)
2. \( (6 - 2) \div (3 + 1) = 2 \)
3. \( 2 + (3 \times 4) = 14 \)
4. \( (5 - 2) \div (3 + 1) = 1 \)

Chapter 1

Order of Operations

Exercise 4

Evaluate each expression using the order of operations.

1. \( 8 \times (5 + 3) = 64 \)
2. \( 12 \div 4 + 2 = 5 \)
3. \( (8 - 2) \times 4 = 24 \)
4. \( 10 \div 2 + 5 = 10 \)

Chapter 1

Formulas

A formula is a mathematical sentence that uses variables to express the relationship between certain quantities. If you know the value of every variable except one unknown variable, you can use the formula to find the value of the unknown variable.

Example

To calculate the number of reams of paper needed to print 172 copies of a 25-page booklet, you can use the formula

\[ n = \frac{r \times 2}{p} \]

where \( n \) is the number of reams needed, \( r \) is a length in centimeters and \( p \) is a mathematical sentence that uses variables to express the relationship between certain quantities.

Exercise

For Exercises 1–3, use the following information.

1. The volume of a sphere is given by the formula \( V = \frac{4}{3} \pi r^3 \), where \( V \) is the volume of the sphere, \( r \) is its radius. What is the volume of the beach ball in cubic centimeters?

2. The volume of a sphere is given by the formula \( V = \frac{4}{3} \pi r^3 \), where \( V \) is the volume of the sphere, \( r \) is its radius. What is the volume of the beach ball in cubic centimeters?

3. Sarah takes 40 breaths to blow up the beach ball. What is the average volume of air per breath?
Chapter 1

NAME ___________________________________________ DATE __________ PERIOD _____

1-1 Skills Practice
Expressions and Formulas

Find the value of each expression.

1. $18 + 2 	imes 3$ 27  
2. $9 + 6 - 2 + 1$ 13  
3. $(3 - 8)(4) - 3$ 97  
4. $5 + 3/2 - 12 + 2$ -7  
5. $-\frac{1}{3}[10 - 9 + 10(3)]$ -7  
6. $\frac{67 - 5}{4}$ 3  
7. $(168 + 73)^2 - 4^3$ 152  
8. $11(5) - 128 - 2^3$ 5 -85

Evaluate each expression if $r = -1, s = 3, t = 12, v = 0,$ and $w = -\frac{1}{2}$.

9. $6r + 2s$ 0  
10. $2vt - 4w$ 64  
11. $u(s - r)$ -2  
12. $s + 2v - 16w$ 1  
13. $(4ab)^2$ 144  
14. $(2v + w)^2$ 3  
15. $(2v + w)^2$ 0  
16. $(2v + w)^2$ 105  
17. $-wv + (t - r)^2$ 26  
18. $(2v + w)^2$ 22

21. TEMPERATURE The formula $K = C + 273$ gives the temperature in kelvins (K) for a given temperature in degrees Celsius. What is the temperature in kelvins when the temperature is 55 degrees Celsius? 328 K

22. TEMPERATURE The formula $C = \frac{5}{9}(F - 32)$ gives the temperature in degrees Celsius for a given temperature in degrees Fahrenheit. What is the temperature in degrees Celsius when the temperature is 68 degrees Fahrenheit? 20°C

23. TEMPERATURE The formula $F = \frac{9}{5}C + 32$ gives the temperature in degrees Fahrenheit for a given temperature in degrees Celsius. What is the temperature in degrees Fahrenheit when the temperature is -40 degrees Celsius? -40°F

24. PHYSICS The formula $h = 120t - 16t^2$ gives the height $h$ in feet of an object $t$ seconds after it is shot upward from Earth's surface with an initial velocity of 120 feet per second. What will the height of the object be after 6 seconds? 144 ft

25. AGRICULTURE Faith owns an organic apple orchard. From her experience the last few seasons, she has developed the formula $P = 20x - 0.01x^2 - 240$ to predict her profit $P$ in dollars this season if her trees produce $x$ bushels of apples. What is Faith's predicted profit this season if her orchard produces 300 bushels of apples? $4860
1. ARRANGEMENTS The chairs in an auditorium are arranged into two rectangles. Both rectangles are 10 rows deep. One rectangle has 6 chairs per row and the other has 12 chairs per row. Write an expression for the total number of chairs in the auditorium.

\[ 10 \times 6 + 10 \times 12 \text{ or } 10(6 + 12) \]

2. GEOMETRY The formula for the area of a ring-shaped object is given by \( A = \pi(R^2 - r^2) \), where \( R \) is the radius of the outer circle and \( r \) is the radius of the inner circle. If \( R = 10 \) inches and \( r = 5 \) inches, what is the area rounded to the nearest square inch?

\[ 236 \text{ in}^2 \]

3. GUESS AND CHECK Amanda received a worksheet from her teacher. Unfortunately, one of the operations in an equation was covered by a blot. What operation is hidden by the blot?

\[ 10 \times 3(4 - 6) = 4 \]

subtraction

4. GAS MILEAGE Rick has \( d \) dollars. The formula for the number of gallons of gasoline that Rick can buy with \( d \) dollars is given by \( g = \frac{d}{3} \). The formula for the number of miles that Rick can drive on \( g \) gallons of gasoline is given by \( m = 21g \). How many miles can Rick drive on \$8 worth of gasoline?

\[ 56 \text{ mi} \]

5. For which speed(s), will you miss the surprise birthday party?

You will miss the party at all speeds 50 mph and less, assuming an 11 A.M. departure.

6. Use your formula to compute the number of minutes it would take to broil a 2 inch thick steak.

\[ 17 \text{ min} \]
**Lesson Reading Guide**

**Properties of Real Numbers**

Get Ready for the Lesson

Read the introduction to Lesson 1-2 in your textbook.
- Why are all of the amounts listed on the register slip at the top of the page followed by negative signs? Sample answer: The amount of each coupon is subtracted from the total amount of purchases so that you save money by using coupons.
- Describe two ways of calculating the amount of money you saved by using coupons if your register slip is the one shown on page 11. Sample answer: Add all the individual coupon amounts or add the amounts for the scanned coupons and multiply the sum by 2.

Read the Lesson

1. Refer to the Key Concepts box on page 11. The numbers 2.57 and 0.010010001… both involve decimals that “go on forever.” Explain why one of these numbers is rational and the other is irrational. Sample answer: 2.57 = 2.5757… is a repeating decimal because there is a block of digits, 57, that repeats forever, so this number is rational. The number 0.010010001… is a non-repeating decimal because, although the digits follow a pattern, there is no block of digits that repeats. So this number is an irrational number.

2. Write the Associative Property of Addition in symbols. Then illustrate this property by finding the sum 12 + 18 + 45 in two different ways. (a + b) + c = a + (b + c); Sample answer: (12 + 18) + 45 = 30 + 45 = 75; 12 + (18 + 45) = 12 + 63 = 75

3. Consider the equations (a · b) · c = a · (b · c) and (a · b) · c = c · (a · b). One of the equations uses the Associative Property of Multiplication and one uses the Commutative Property of Multiplication. How can you tell which property is being used in each equation? The first equation uses the Associative Property of Multiplication. The second equation uses the Commutative Property of Multiplication.

Remember What You Learned

4. How can the meanings of the words **commuter** and **association** help you to remember the difference between the commutative and associative properties? Sample answer: A commuter is someone who travels back and forth to work or another place, and the commutative property says you can switch the order when two numbers that are being added or multiplied. An association is a group of people who are connected or united, and the associative property says that you can switch the grouping when three numbers are added or multiplied.

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**Study Guide and Intervention**

**Properties of Real Numbers**

Real Numbers All real numbers can be classified as either rational or irrational. The set of rational numbers includes several subsets: natural numbers, whole numbers, and integers.

- **R** real numbers (all rationals and irrationals)
- **Q** rational numbers (all numbers that can be represented in the form \( \frac{m}{n} \); where m and n are integers and n is not equal to 0)
- **I** irrational numbers (all nonterminating, nonrepeating decimals)
- **N** natural numbers (1, 2, 3, 4, 5, 6, 7, 8, …)
- **W** whole numbers (0, 1, 2, 3, 4, 5, 6, 7, 8, …)
- **Z** integers (…, -3, -2, -1, 0, 1, 2, 3, …)

**Example**

Name the sets of numbers to which each number belongs.

a. \( \frac{11}{3} \) rationals (Q), reals (R)

b. \( \sqrt{25} \) naturals (N), wholes (W), integers (Z), rationals (Q), reals (R)

**Exercises**

Name the sets of numbers to which each number belongs.

1. \( \frac{6}{7} \) Q, R

2. \( -\sqrt{81} \) Z, Q, R

3. 0 W, Z, Q, R

4. 192.0005 Q, R

5. 73 N, W, Z, Q, R

6. 34 \( \frac{1}{2} \) Q, R

7. \( \sqrt{\frac{36}{9}} \) Q, R

8. 26.1 Q, R

9. \( \pm 1 \) R

10. \( \frac{15}{3} \) N, W, Z, Q, R

11. \( -4.17 \) Q, R

12. \( \frac{25}{5} \) N, W, Z, Q, R

13. \( -1 \) Z, Q, R

14. \( \sqrt{42} \) I, R

15. \( -11.2 \) Q, R

16. \( -\frac{8}{13} \) Q, R

17. \( \frac{\sqrt{2}}{2} \) I, R

18. 33.3 Q, R

19. 894,000 N, W, Z, Q, R

20. -0.02 Q, R
1.2 Properties of Real Numbers

**Example**

Simplify $9x + 3y + 12y - 0.9x$.

$9x + 3y + 12y - 0.9x = 9x + (-0.9x) + 3y + 12y$

Commutative Property ($+$)

$= (9 + (-0.9))x + (3 + 12)y$

Distributive Property

$= 8.1x + 15y$

**Exercises**

Simplify each expression.

1. $3a - b + 4(2b - a)$
2. $40x + 18x - 5x + 11s$
3. $\frac{1}{2}(4j + 2k - 6x + 3k)$

**Answers (Lesson 1-2)**

1. $3a - b + 4(2b - a)$
2. $40x + 18x - 5x + 11s$
3. $\frac{1}{2}(4j + 2k - 6x + 3k)$

**Skills Practice**

**Properties of Real Numbers**

Name the sets of numbers to which each number belongs.

1. $34 \quad \mathbb{N}, \mathbb{W}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$
2. $-525 \quad \mathbb{Z}, \mathbb{Q}, \mathbb{R}$
3. $0.875 \quad \mathbb{Q}, \mathbb{R}$
4. $\frac{12}{3} \quad \mathbb{N}, \mathbb{W}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$
5. $-\sqrt{9} \quad \mathbb{Z}, \mathbb{Q}, \mathbb{R}$
6. $\sqrt{30} \quad \mathbb{I}, \mathbb{R}$

Name the property illustrated by each equation.

7. $3 \cdot x = x \cdot 3$ Comm. ($\cdot$)
8. $3q + 0 = 3q$ Add. Iden.
9. $2r + w = 2r + 2w$ Distributive
10. $(3r + 4r) = (2r + 3r) + 4r$ Assoc. ($+$)
11. $\frac{1}{5} \cdot \left(\frac{1}{6}\right) = 1$ Mult. Inv.
12. $15 \cdot (1) = 15\mathbf{c}$ Mult. Iden.
13. $0.625(0.5) = 0.625(0.5)$ Assoc. ($\cdot$)
14. $(10 + 12b) + 7b = (12b + 10b) + 7b$ Comm. ($+$)

Name the additive inverse and multiplicative inverse for each number.

15. $15 = -15, \frac{1}{15}$
16. $1.25 = -1.25, 0.8$
17. $\frac{4}{5} = -\frac{5}{4}, \frac{4}{5}$
18. $\frac{3}{4} = -\frac{3}{4}, \frac{4}{15}$

Simplify each expression.

19. $3r + 5 + 2x - 3 = 5x + 2$
20. $x - y - z + y - x + z = 0$
21. $-(3g + 3h) + 5g - 10h = 2g - 13h$
22. $a^2 - a + 4a - 3a^2 + 1 - 2a^2 + 3a + 1$
23. $3m - z + 5 + 2m - z = 13m - 8z$
24. $2x - 3y + (5x + 5y - 5x - 2z) - 3x + 2z$
25. $6(2 + 8) - 4(2b + 1) = 8 - 2v$
26. $\frac{1}{3}(15d + 3) - \frac{1}{3}(8 - 10d) = 10d - 3$

Chapter 1

Glencoe Algebra 2
1. MENTAL MATH When teaching elementary students to multiply and learn place value, books often show that $54 \times 8 = (50 + 4) \times 8 = (50 \times 8) + (4 \times 8)$. What property is used? **Distributive Property**

2. MODELS What property of real numbers is illustrated by the figure below? **Commutative Property of Multiplication**

3. VENN DIAGRAMS Make a Venn diagram that shows the relationship between natural numbers, integers, rational numbers, irrational numbers, and real numbers.

4. NUMBER THEORY Consider the following two statements.
   I. The product of any two rational numbers is always another rational number.
   II. The product of two irrational numbers is always irrational.
   Determine if these statements are always, sometimes, or never true. Explain.
   I. always
   II. sometimes, \( \sqrt{2} \times \sqrt{2} = 2 \)

RIGHT TRIANGLES For Exercises 5–7, use the following information.

The lengths of the sides of the right triangle shown are related by the formula \( c^2 = a^2 + b^2 \).

For each set of values of \( a \) and \( b \), determine the value of \( c \). State whether \( c \) is a natural number.

5. \( a = 5, b = 12 \)
   \( c = 13; \) it is a natural number.

6. \( a = 7, b = 14 \)
   \( c = \sqrt{245} \) or \( 7\sqrt{5} \); it is not a natural number.

7. \( a = 7, b = 24 \)
   \( c = 25; \) it is a natural number.

1. MENTAL MATH

   - When teaching elementary students to multiply and learn place value, books often show that $54 \times 8 = (50 + 4) \times 8 = (50 \times 8) + (4 \times 8)$. What property is used? **Distributive Property**

2. MODELS

   - What property of real numbers is illustrated by the figure below? **Commutative Property of Multiplication**

3. VENN DIAGRAMS

   - Make a Venn diagram that shows the relationship between natural numbers, integers, rational numbers, irrational numbers, and real numbers.

4. NUMBER THEORY

   - Consider the following two statements.
     I. The product of any two rational numbers is always another rational number.
     II. The product of two irrational numbers is always irrational.
   - Determine if these statements are always, sometimes, or never true. Explain.
     I. always
     II. sometimes, \( \sqrt{2} \times \sqrt{2} = 2 \)

RIGHT TRIANGLES

- For Exercises 5–7, use the following information.

   - The lengths of the sides of the right triangle shown are related by the formula \( c^2 = a^2 + b^2 \).
   - For each set of values of \( a \) and \( b \), determine the value of \( c \). State whether \( c \) is a natural number.
     5. \( a = 5, b = 12 \)
        \( c = 13; \) it is a natural number.
     6. \( a = 7, b = 14 \)
        \( c = \sqrt{245} \) or \( 7\sqrt{5} \); it is not a natural number.
     7. \( a = 7, b = 24 \)
        \( c = 25; \) it is a natural number.
Properties of a Group

A set of numbers forms a group with respect to an operation if for that operation the set has (1) the Closure Property, (2) the Associative Property, (3) a member which is an identity, and (4) an inverse for each member of the set.

**Example 1**  
Does the set \{0, 1, 2, 3, ...\} form a group with respect to addition?

**Closure Property:** For all numbers in the set, is \(a + b\) in the set? \(0 + 1 = 1\), and 1 is in the set; \(0 + 2 = 2\), and 2 is in the set; and so on. The set has closure for addition.

**Associative Property:** For all numbers in the set, does \(a + (b + c) = (a + b) + c\)?  
\(0 + (1 + 2) = (0 + 1) + 2; 1 + (2 + 3) = (1 + 2) + 3\; and\; so\; on.\) The set is associative for addition.

**Identity:** Is there some number, \(i\), in the set such that \(i + a = a + i = a\) for all \(a\)? \(0 + 1 = 1 + 0 = 1\; and\; so\; on.\) The identity for addition is 0.

**Inverse:** Does each number, \(a\), have an inverse, \(a'\), such that \(a + a' = a' + a = i\)? The integer inverse of 3 is \(-3\) since \(-3 + 3 = 0\), and 0 is the identity. But the set does not contain \(-3\). Therefore, there is no inverse for 3.

The set is not a group with respect to addition because only three of the four properties hold.

**Example 2**  
Is the set \{-1, 1\} a group with respect to multiplication?

**Closure Property:** \((-1)(-1) = 1; (-1)(1) = -1; (1)(-1) = -1; (1)(1) = 1\)  
The set has closure for multiplication.

**Associative Property:** \((-1)(-1)(-1) = (-1)(1) = -1\; and\; so\; on.\)  
The set is associative for multiplication.

**Identity:** \(1(-1) = -1; 1(1) = 1\)  
The identity for multiplication is 1.

**Inverse:** -1 is the inverse of -1, since \((-1)(-1) = 1\), and 1 is the identity.  
1 is the inverse of 1 since \((1)(1) = 1\), and 1 is the identity.  
Each member has an inverse.

The set \{-1, 1\} is a group with respect to multiplication because all four properties hold.

**Exercise**

Tell whether the set forms a group with respect to the given operation.

1. Integers, addition yes  
2. Integers, multiplication no  
3. \{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots\}, addition no  
4. Multiples of 5, multiplication no  
5. \{x, x^2, x^3, x^4, \ldots\}, addition no  
6. \{\sqrt{3}, \sqrt{2}, \sqrt{\frac{1}{2}}, \ldots\}, multiplication no  
7. Irrational numbers, addition no  
8. Irrational numbers, addition yes

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Chapter 1

1-2 Enrichment

1-3 Lesson Reading Guide

Solving Equations

Get Ready for the Lesson

Read the introduction to Lesson 1-3 in your textbook.

- To find your target heart rate, what two pieces of information must you supply? age (A) and desired intensity level (I)
- Write an equation that shows how to calculate your target heart rate.  
  \[P = \frac{(220 - A) \cdot I}{6}\]

Read the Lesson

1. a. How are algebraic expressions and equations alike?

   Sample answer: Both contain variables, constants, and operation signs.

   b. How are algebraic expressions and equations different?

   Sample answer: Equations contain equal signs; expressions do not.

   c. How are algebraic expressions and equations related?

   Sample answer: An equation is a statement that says that two algebraic expressions are equal.

Read the following problem and then write an equation that you could use to solve it. Do not actually solve the equation. In your equation, let \(m\) be the number of miles driven.

2. When Louisa rented a moving truck, she agreed to pay $28 per day plus $0.42 per mile. If she kept the truck for 3 days and the rental charges (without tax) were $153.72, how many miles did Louisa drive the truck?  
   \[(28)(3) + 0.42m = 153.72\]

Remember What You Learned

3. How can the words reflection and symmetry help you remember and distinguish between the reflexive and symmetric properties of equality? Think about how these words are used in everyday life or in geometry.

Sample answer: When you look at your reflection, you are looking at yourself. The reflexive property says that every number is equal to itself. In geometry, symmetry with respect to a line means that the parts of a figure on the two sides of a line are identical. The symmetric property of equality allows you to interchange the two sides of an equation. The equal sign is like the line of symmetry.
**Study Guide and Intervention**

**Solving Equations**

**Verbal Expressions to Algebraic Expressions**

The chart suggests some ways to help you translate word expressions into algebraic expressions. Any letter can be used to represent a number that is not known.

<table>
<thead>
<tr>
<th>Word Expression</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>sum of two numbers</td>
<td>addition</td>
</tr>
<tr>
<td>difference of two numbers</td>
<td>subtraction</td>
</tr>
<tr>
<td>product of two numbers</td>
<td>multiplication</td>
</tr>
<tr>
<td>quotient of two numbers</td>
<td>division</td>
</tr>
</tbody>
</table>

**Example 1**

Write an algebraic expression to represent 18 less than the quotient of a number and 3.

\[
\frac{n}{3} - 18
\]

**Example 2**

Write a verbal sentence to represent \(6(n - 2) = 14\).

Six times the difference of a number and two is equal to 14.

**Exercises**

Write an algebraic expression to represent each verbal expression.

1. the sum of six times a number and 25
   \[6n + 25\]

2. four times the sum of a number and 3
   \[4(n + 3)\]

3. 7 less than fifteen times a number
   \[15n - 7\]

4. the difference of nine times a number and the quotient of 6 and the same number
   \[9n - \frac{6}{n}\]

5. the sum of 100 and four times a number
   \[100 + 4n\]

6. the product of 3 and the sum of 11 and a number
   \[3(11 + n)\]

7. four times the square of a number increased by five times the same number
   \[4n^2 + 5n\]

8. 23 more than the product of 7 and a number
   \[7n + 23\]

Write a verbal sentence to represent each equation. Sample answers are given.

9. \(3n - 35 = 79\)
   The difference of three times a number and 35 equals 79.

10. \(3n^3 + 3n^2 - 4n\)
   Twice the sum of the cube of a number and three times the square of the number is equal to four times the number.

11. \(\frac{5n}{n} - \frac{n}{3} - n - 8\)
   The quotient of five times a number and the sum of the number and 3 is equal to the difference of the number and 8.

Solve each equation. Check your solution.

1. \(3x = 45\)
   \[x = \frac{45}{3} = 15\]

2. \(17 = 9 - a\)
   \[a = 9 - 17 = -8\]

3. \(5x - 1 = 6x - 5\)
   \[x = 5 + 5 = 10\]

4. \(\frac{3}{2}m = \frac{1}{2} + 3\)
   \[m = \frac{1}{2} + 3 = \frac{7}{2}\]

5. \(7 - \frac{3}{2}x = 3\)
   \[x = \frac{7 - 3}{\frac{3}{2}} = \frac{2}{3}\]

6. \(-68 = -2(e + 7) - 3\)
   \[e = \frac{-68 + 3}{-2} = 32.5\]

7. \(0.25b - 10 = 50\)
   \[b = \frac{50 + 10}{0.25} = 240\]

8. \(3x + 17 = 5x - 13\)
   \[x = \frac{17 + 13}{2} = 15\]

9. \(5(4 - k) = -10k - 4\)
   \[k = \frac{5}{4} \cdot \frac{4 - k}{-10k} = \frac{-5}{4}\]

10. \(120 - \frac{3}{4}q = 60\)
    \[q = \frac{120 - 60}{\frac{3}{4}} = 160\]

11. \(3n = 98 - 16\)
    \[n = \frac{98 - 16}{3} = 24\]

12. \(4.5 + 2p = 87.21\)
    \[p = \frac{87.21 - 4.5}{2} = 38.855\]

13. \(4n + 20 = 53 - 2n\)
    \[n = \frac{53 - 20 - 2n}{4} = 4\]

14. \(100 - 20 = 5n - 16\)
    \[n = \frac{100 - 20 + 16}{5} = 20\]

15. \(12x + 75 = 102 - x\)
    \[x = \frac{102 - 75}{12 + 1} = 2\]

Solve each equation or formula for the specified variable.

16. \(a = 3b - c, \text{ for } b = \frac{a + c}{3}\)
    \[b = \frac{a + c}{3}\]

17. \(y = 10, \text{ for } t = \frac{8}{3}\)
    \[t = \frac{8}{3} \times 10 = 26.67\]

18. \(h = 12r - 1, \text{ for } g = \frac{h + 1}{12}\)
    \[g = \frac{12r - 1 + 1}{12} = \frac{12r}{12} = r\]

19. \(\frac{2p}{q} = 12, \text{ for } p = \frac{4r}{q}\)
    \[p = \frac{4r}{q} \times 12 = \frac{48r}{q}\]

20. \(2cy = x + 7, \text{ for } x = \frac{7}{2y} - 1\)
    \[x = \frac{7}{2y} - 1\]

21. \(d = \frac{f}{2} + 6, \text{ for } f = \frac{24 - 2d}{2}\)
    \[f = \frac{24 - 2d}{2} \times 2 = 12 - d\]

22. \(3y - h = 108, \text{ for } j = \frac{18 + k}{2}\)
    \[j = \frac{18 + k}{2}\]

23. \(3.5s = 42 - 14t, \text{ for } x = 4t + 12\)
    \[x = 4t + 12\]

24. \(m + 5n = 20, \text{ for } m = \frac{20n}{5n + 1}\)
    \[m = \frac{20n}{5n + 1}\]

25. \(4x - 3y = 10, \text{ for } y = \frac{4x - 10}{3}\)
    \[y = \frac{4x - 10}{3}\]
Write an algebraic expression to represent each verbal expression.

1. 4 times a number, increased by 7
   \[ 4n + 7 \]
2. 8 less than 5 times a number
   \[ 5n - 8 \]
3. 6 times the sum of a number and 5
   \[ 6(n + 5) \]
4. The product of 3 and a number, divided by 9
   \[ \frac{3n}{9} \]
5. 3 times the difference of 4 and a number
   \[ 3(4 - n) \]
6. The product of –11 and the square of a number
   \[ -11n^2 \]

Write a verbal expression to represent each equation.

7. \[ n + 8 = 16 \]
   The difference of a number and 8 is 16.
8. \[ 8 + 3x = 5 \]
   The sum of 8 and 3 times a number is 5.
9. \[ b^2 + 3 = b \]
   Three added to the square of a number is the number.
10. \[ \frac{2}{3} = 2 - 2y \]
    A number divided by 3 is the difference of 2 and twice the number.

Name the property illustrated by each statement.

11. If \( a = 0.5b \), and 0.5b = 10, then \( a = 10 \).
    Transitive (\(\implies\))
12. If \( d + 1 = f \), then \( d = f - 1 \).
    Subtraction (\(\implies\))
13. If \( -7c = 14 \), then \( 14 = -7c \).
    Symmetric (\(\implies\))
14. If \( 8 + 7x = 30 \), then \( 15x = 30 \).
    Substitution (\(\implies\))

Solve each equation. Check your solution.

15. \[ 4n + 2 = 18 \]
16. \[ x + 4 = 5x + 2 \]
17. \[ 3x + 2x = 15 \]
18. \[ -3b + 7 = 15 + 2b \]
19. \[ -5x - 3x = 24 \]
20. \[ 4e + 20 - 6 = 3d \]
21. \[ a - \frac{2a}{5} = 3 \]
22. \[ 2.2n + 0.8n + 5 = 4n \]

Solve each equation or formula for the specified variable.

23. \( I = prt \), for \( p \)
    \[ p = \frac{I}{rt} \]
24. \[ y = \frac{1}{4}x - 12 \], for \( x \)
    \[ x = 4y + 48 \]
25. \[ A = \frac{x^2 + y}{2} \], for \( y \)
    \[ y = 2A - x \]
26. \[ A = 2\pi^2 + 2\pi h \], for \( h \)
    \[ h = \frac{A - 2\pi^2}{2\pi} \]

Write an algebraic expression to represent each verbal expression.

1. 2 more than the quotient of a number and 5
   \[ \frac{n}{5} + 2 \]
2. The sum of two consecutive integers
   \[ n + (n + 1) \]
3. 5 times the sum of a number and 1
   \[ 5(m + 1) \]
4. 1 less than twice the square of a number
   \[ 2y^2 - 1 \]

Write a verbal expression to represent each equation. 5–8. Sample answers are given.

5. \[ 5 - 2x = 4 \]
   The difference of 5 and twice a number is 4.
6. \[ 3y - 4y^3 \]
   Three times a number is 4 times the cube of the number.
7. \[ 3c - 2(c - 1) \]
   Three times a number is twice the difference of the number and 1.
8. \[ \frac{c}{5} = 3 - 2m + 1 \]
   The quotient of a number and 5 is 3 times the sum of twice the number.

Name the property illustrated by each statement.

9. If \( t - 13 = 52 \), then \( 52 = t - 13 \).
    Symmetric (\(\implies\))
10. If \( 82q + 1 = 4 \), then \( 2q + 1 = 1 \).
    Division (\(\implies\))
11. If \( h + 12 = 22 \), then \( h = 10 \).
    Subtraction (\(\implies\))
12. If \( 4m = -15 \), then \( -12m = 45 \).
    Multiplication (\(\implies\))

Solve each equation. Check your solution.

13. \[ 14 = 8 - 6v - 1 \]
14. \[ 9 + 4n = -59 - 17 \]
15. \[ \frac{3}{4} = \frac{1}{2}n \]
16. \[ \frac{5}{6} + \frac{3}{4} = \frac{11}{12} \]
17. \[ -1.6r + 5 = -7.8 \]
18. \[ 6x - 5 = 7 - 9x + \frac{4}{5} \]
19. \[ 5(4 - 4x) = 21 + \frac{3}{2} \]
20. \[ 6y - 5 = -32y + 1 \]

Solve each equation or formula for the specified variable.

21. \[ E = mc^2 \], for \( m \)
    \[ m = \frac{E}{c^2} \]
22. \[ c = \frac{2d + 1}{3} \], for \( d \)
    \[ d = \frac{3c - 1}{2} \]
23. \[ h = vt - \frac{1}{2}gt^2 \], for \( v \)
    \[ v = \frac{h + gt^2}{t} \]
24. \[ E = \frac{1}{2}Iw^2 + U \], for \( I \)
    \[ I = \frac{2(E - U)}{w^2} \]

Define a variable, write an equation, and solve the problem.

25. GEOMETRY The length of a rectangle is twice the width. Find the width if the perimeter is 60 centimeters.
    \[ w = \text{width}; 2(2w) + 2w = 60; 10 \text{ cm} \]
26. GOLF Luis and three friends went golfing. Two of the friends rented clubs for $86 each. The total cost of the rented clubs and the green fees for each person was $76. What was the cost of the green fees for each person?
    \[ g = \text{green fees per person}; 6(2) + 4g = 76; \]$16
1. AGES Robert's father is 5 years older than 3 times Robert's age. Let Robert's age be denoted by \( R \) and let Robert's father's age be denoted by \( F \). Write an equation that relates Robert's age and his father's age. \( F = 3R + 5 \).

2. AIRPLANES The number of passengers \( p \) and the number of suitcases \( s \) that an airplane can carry are related by the equation \( 180p + 60s = 3,000 \). If 10 people board the aircraft, how many suitcases can the airplane carry? 20 suitcases.

3. GEOMETRY The length of a rectangle is 10 units longer than its width. If the total perimeter of the rectangle is 44 units, what is the width? \( w = 6 \) units.

4. SAVINGS Jason started with \( d \) dollars in his piggy bank. One week later, Jason doubled the amount in his piggy bank. Another week later, Jason was able to add \$20 to his piggy bank. At this point, the piggy bank had \$50 in it. What is \( d \)? 15.

5. DOMINOES For Exercises 5 and 6, use the information below.

   Nancy is setting up a train of dominoes from the front entrance straight down the hall to the kitchen entrance. The thickness of each domino is \( t \). Nancy places the dominoes so that the space separating consecutive dominoes is \( 3t \). The total distance that \( N \) dominoes takes up is given by \( d = (4N + 1)t \).

   \[ d = 321 \text{ centimeters} \]

   6. How many dominoes did Nancy have in her hallway? 80 dominoes.

6. The Gross National Product, GNP, is an important indicator of U.S. economy. The GNP contains information about the inflation rate, the Bond market, and the Stock market. It is composed of consumer goods, investments, government expenditures, exports, and imports.

   \[ \text{GNP} = C + I + G + X - M \]

   where

   - \( C \) is consumer goods (e.g. TV's, Cars, Food, Furniture, Clothes, Doctors' fees, and Dining)
   - \( I \) is investments (e.g. Factories, Computers, Airlines, and Housing)
   - \( G \) is government spending and investments (e.g. Ships, Roads, Schools, NASA, and Bombs)
   - \( X \) is exports (e.g. Corn, Wheat, Cars, and Computers)
   - \( M \) is for imports (e.g. Cars, Computer chips, Clothes, and Oil)

1. The most important sector of the U.S. economy is consumption. It makes up about 60% of the entire GNP. In 2000, the U.S.'s GNP was 10.5 trillion dollars. In the same year, there were 1 trillion dollars in investments, but a 1 trillion dollar trade deficit. Assuming that consumption made up 60% of the GNP, how much did the government budget for spending?

   \[ \text{Government budget} = 0.6 \times 10.5 \text{ trillion} = 6.3 \text{ trillion} \]

   What might have caused the change?

   Government spending must have gone up, in fact \( G = 6 \) trillion dollars. Employment may have caused a dip in consumption.

2. In 2001, the U.S. trade deficit remain at 1 trillion dollars, investments also remain steady at 1 trillion dollars. However consumption dipped to only 50% of the GNP, which increased to 12 trillion dollars. What was the effect on government spending?

   Employment may have caused a dip in consumption.

3. If the GNP remains steady, and so do investments and government spending, but the trade deficit increases (to say 2 or 3 trillion dollars), what does this say about the consumption level?

   Consumption will increase.

4. Determine if there is a trade surplus or deficit when there is 12 trillion dollar GNP, 2 trillion in investments, 3 trillion in government investments, and 5 trillion in consumption. Explain why this situation may be favorable. There is a surplus. Consumption is reduced, possibly because the cost of living has reduced.
Chapter 1

NAME ______________________________________________ DATE______________ PERIOD _____

Graphing Calculator Activity

1-3 Solving Equations and Checking Solutions

When solving equations, checking the solutions is an important process. A graphing calculator can be used to check the solution of an equation.

Example 1

Solve $-2(5y - 1) - y = -4(y - 3)$.

Graph the expression on the left side of the equation in Y1 and the expression on the right side of the equation in Y2. Choose an appropriate view window so that the intersection of the graphs is visible. Then use the intersect command to find the coordinates of the common point.


The x-coordinate, $-\frac{10}{7}$, is the solution to the equation. The y-coordinate is the value of both sides of the equation when $x = -\frac{10}{7}$.

Example 2

Solve $\frac{x}{2} - \frac{y}{3} = \frac{1}{2} (y - 2)$.

Graph the expression on the left side of the equation in Y1 and the expression on the right side of the equation in Y2. Enter Y1 - Y2 in Y3. Then graph the function in Y3. Use the zero function under the CALC menu to determine where the graph of Y3 equals zero. This point will be the solution.


Use arrow keys and enter to set the bound prompts. The solution is $x = \frac{20}{11}$.

Exercises

Solve each equation.

1. $-3(2w - 7) = 9 - 25w + 4$; $\quad w = -\frac{3}{7}$

2. $1.5(x - x) = 1.3(2 - x)$; $\quad x = 17$

3. $\frac{1}{2} (a + 2) = \frac{1}{3} (5 - a)$; $\quad a = \frac{4}{5}$

4. $\frac{3}{2}(x + 25) - 2x - 11 = 78$; $\quad x = \frac{1}{4}$

5. $m - 4 \quad \frac{3m - 1}{5} = 1$; $\quad m = -8$

6. $x + 5 + \frac{1}{2} = 2x - x - \frac{3}{8}$; $\quad x = \frac{21}{11}$

Answers

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Chapter 1

NAME ______________________________________________ DATE______________ PERIOD _____

Lesson Reading Guide

1-4 Solving Absolute Value Equations

Get Ready for the Lesson

Read the introduction to Lesson 1-4 in your textbook.

- What is a seismologist and what does magnitude of an earthquake mean? a scientist who studies earthquakes; a number from 1 to 10 that tells how strong an earthquake is

- Why is an absolute value equation rather than an equation without absolute value used to find the extremes in the actual magnitude of an earthquake in relation to its measured value on the Richter scale? Sample answer: The actual magnitude can vary from the measured magnitude by up to 0.3 unit in either direction, so an absolute value equation is needed.

- If the magnitude of an earthquake is estimated to be 6.9 on the Richter scale, it might actually have a magnitude as low as or as high as 6.6 or as high as 7.2.

Read the Lesson

1. Explain how $-a$ could represent a positive number. Give an example. Sample answer: If $a$ is negative, then $-a$ is positive. Example: If $a = -25$, then $-a = 25$.

2. Explain why the absolute value of a number can never be negative. Sample answer: The absolute value is the number of units it is from 0 on the number line. The number of units is never negative.

3. What does the sentence $b = 0$ mean? Sample answer: The number $b$ is 0 or greater than 0.

4. What does the symbol $\bigcirc$ mean as a solution set? Sample answer: If a solution set is $\bigcirc$, then there are no solutions.

Remember What You Learned

5. How can the number line model for absolute value that is shown on page 28 of your textbook help you remember that many absolute value equations have two solutions? Sample answer: The number line shows that for every positive number, there are two numbers that have that number as their absolute value.
Example 1
Evaluate \(|-4| - |-2x|\) if \(x = 6\).

\[
|-4| - |-2x| = |-4| - |-12| = 4 - 12 = -8
\]

Example 2
Evaluate \(|2x - 3y|\) if \(x = -4\) and \(y = 3\).

\[
|2(-4) - 3(3)| = |-8 - 9| = 17
\]

Exercise
Evaluate each expression if \(x = -4\), \(x = 2\), \(y = \frac{3}{2}\), and \(z = -6\).

1. \(|2x - 8|\)
2. \(|x + z| - |-7|\)
3. \(5 + |w + z|\)
4. \(|x + 5| - |2w|\)
5. \(|x| - |y| - |z| - \frac{4}{2}\)
6. \(|7 - x| + |3x|\)
7. \(|w - 4x|\)
8. \(|wz| - |xy|\)
9. \(|z| - |3yz|\)
10. \(|5w| + |2x - 2y|\)
11. \(|x| - 4|2x + y|\)
12. \(10 - |xw|\)
13. \(|6y + z| + |x|\)
14. \(3|xw| + \frac{1}{2}|4x + 8y|\)
15. \(17|z|x - 30 - 9\)
16. \(|14 - 2w - xy|\)
17. \(|2x - y| + 5y\)
18. \(|xyz| + |wz|\)
19. \(|z| + |x| + |x| - 32\)
20. \(12 - |10x - 10y|\)
21. \(\frac{1}{2}|5z + 8v|\)
22. \(|yz - 4w| - |w + 17|\)

23. \(\frac{3}{4}|wz| + \frac{1}{2}|8y|\)
24. \(2x + |x|\)

CHECK
\(|2x - 3|\) = 17
\(17 - 17\)

There are two solutions, 10 and -7.

Exercises
Solve each equation. Check your solutions.

1. \(|x + 15| = 37\)
2. \(|t - 4| = 5 = 0\)
3. \(|x - 5| = 45\)
4. \(|m + 3| = 12 - 2m\)
5. \(|5b + 9| + 16 = 2\)
6. \(|15 - 2b| = 45\)
7. \(|5e + 24 = 8 - 3n|\)
8. \(|8 + 5e| = 14 - a\)
9. \(\frac{1}{2}|4p - 11| + p = 4\)
10. \(|3x - 1| = 2e + 11\)
11. \(\frac{1}{3}|x + 3| = -1\)
12. \(40 - 4x - 2|3e - 10|\)
13. \(5' - |5' + 4| = 20\)
14. \(|4b + 3| = 15 - 2b\)
15. \(\frac{1}{2}|16 - 2b| = -3x + 1\)
16. \(16 - 3x - 4x = 12\)

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### 1-4 Skills Practice

**Solving Absolute Value Equations**

Evaluate each expression if \( w = \frac{2}{3} \), \( x = \frac{2}{3} \), \( y = -\frac{2}{3} \), and \( z = -\frac{1}{2} \).

<table>
<thead>
<tr>
<th>Expression</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>5w</td>
</tr>
<tr>
<td>(</td>
<td>-9y</td>
</tr>
<tr>
<td>(</td>
<td>9y - z</td>
</tr>
<tr>
<td>(</td>
<td>-10z - 31</td>
</tr>
<tr>
<td>(</td>
<td>5x + 1</td>
</tr>
<tr>
<td>(</td>
<td>4w - 3.2</td>
</tr>
<tr>
<td>(</td>
<td>-3x - 2y</td>
</tr>
<tr>
<td>(</td>
<td>w - 1</td>
</tr>
</tbody>
</table>

Solve each equation. Check your solutions.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>y + 3</td>
</tr>
<tr>
<td>(</td>
<td>3x - 6</td>
</tr>
<tr>
<td>(</td>
<td>2x + 1</td>
</tr>
<tr>
<td>(</td>
<td>p - 7</td>
</tr>
<tr>
<td>(</td>
<td>7 - y</td>
</tr>
<tr>
<td>(\frac{1}{3}(9,\frac{5}{2}))</td>
<td>({-2, 2})</td>
</tr>
<tr>
<td>(\frac{1}{2}(5,\frac{1}{6}))</td>
<td>({-2, 2})</td>
</tr>
<tr>
<td>(\frac{5}{6} + 2)</td>
<td>(\emptyset)</td>
</tr>
<tr>
<td>(</td>
<td>x + 10</td>
</tr>
</tbody>
</table>

29. **WEATHER**

A thermometer comes with a guarantee that the stated temperature differs from the actual temperature by no more than 1.5 degrees Fahrenheit. Write and solve an equation to find the minimum and maximum actual temperatures when the thermometer states that the temperature is 87.4 degrees Fahrenheit.

30. **OPINION POLLS**

Public opinion polls reported in newspapers are usually given with a margin of error. For example, a poll with a margin of error of ±5% is considered accurate to within plus or minus 5% of the actual value. A poll with a stated margin of error of ±5% predicts that candidate Tonwe will receive 51% of an upcoming vote. Write and solve an equation describing the minimum and maximum percent of the vote that candidate Tonwe is expected to receive.

\[|x - 51| \leq 3 \text{ or } 48 \leq x \leq 54\]
1. LOCATIONS Identical vacation cottages, equally spaced along a street, are numbered consecutively beginning with 10. Maria lives in cottage #17. Joshua lives 4 cottages away from Maria. If n represents Joshua’s cottage number, then \( |n - 17| = 4 \). What are the possible numbers of Joshua’s cottage?

2. HEIGHT Sarah and Jessica are sisters. Sarah’s height is \( x \) inches and Jessica’s height is \( y \) inches. Their father wants to know how many inches separate them. Write an equation to describe this difference in such a way that the result will always be positive no matter which sister is taller. 

3. AGES Rhonda conducts a survey of the ages of students in eleventh grade at her school. On November 1, she finds the average age is 160 months. She also finds that two-thirds of the students are within 3 months of the average age. Write and solve an equation to determine the age limits for this group of students.

\[ |a - 200| = 3 \]
\[ a = 197 \text{ or } 203 \]

4. TOLERANCE Martin makes exercise weights. For his 10 pound dumbbells, he guarantees that the actual weight of his dumbbells is within 0.1 pounds of 10 pounds. Write and solve an equation that describes the minimum and maximum weight of his 10 pound dumbbells.

\[ |w - 10| = 0.1 \]
minimum weight: 9.9 pounds
maximum weight: 10.1 pounds

WALKING For Exercises 5-7, use the following information.

Jim is walking along a straight line. An observer watches him. If Jim walks forward, the observer records the distance as a positive number, but if he walks backward, the observer records the distance as a negative number. The observer has recorded that Jim has walked \( a \) feet, then \( b \) feet, then \( c \) feet.

5. Write a formula for the total distance that Jim walked.

\[ T = |a| + |b| + |c| \]

6. The equation you wrote in part A should not be \( T = |a + b + c| \). What does \( |a + b + c| \) represent? The distance Jim ends up from where he started.

7. When would the formula you wrote in part A give the same value as the formula shown in part B? They will be equal only if Jim walks in the same direction each time giving \( a \), \( b \), and \( c \) all the same sign.

Considering All Cases in Absolute Value Equations

You have learned that absolute value equations with one set of absolute value symbols have two cases that must be considered. For example, \( |x + 3| = 5 \) must be broken into \( x + 3 = 5 \) or \( -(x + 3) = 5 \). For an equation with two sets of absolute value symbols, four cases must be considered.

Consider the problem \( |x + 2| + 3 = |x + 6| \). First we must write the equations for the case where \( x + 2 \geq 0 \) and where \( x + 2 < 0 \). Here are the equations for these two cases:

\[ x + 2 + 3 = x + 6 \]
\[ x + 2 + 3 = -(x + 6) \]
\[ -(x + 2) + 3 = x + 6 \]
\[ -(x + 2) + 3 = -(x + 6) \]

Each of these equations also has two cases. By writing the equations for both cases of each equation above, you end up with the following four equations:

\[ x + 2 + 3 = x + 6 \]
\[ x + 2 + 3 = -(x + 6) \]
\[ -(x + 2) + 3 = x + 6 \]
\[ -(x + 2) + 3 = -(x + 6) \]

Solve each of these equations and check your solutions in the original equation, \( |x + 2| + 3 = |x + 6| \). The only solution to this equation is \( x = 2 \).

Exercises

Solve each absolute value equation. Check your solution.

1. \( |x - 4| = |x + 7| \) \( x = -1.5 \)
2. \( |2x + 9| = |x - 3| \) \( x = -12 \), \( -2 \)
3. \( |x - 6| = |5x + 10| \) \( x = -2 \)
4. \( |x + 4| - 6 = |x - 3| \) \( x = 2.5 \)

5. How many cases would there be for an absolute value equation containing three sets of absolute value symbols? 8

6. List each case and solve \( |x + 2| + |2x - 4| = |x - 3| \). Check your solution.

\[ x + 2 + 2x - 4 = x - 3 \]
\[ -x + 2 - 2x - 4 = -(x - 3) \]
\[ x + 2 + (2x - 4) = x - 3 \]
\[ x + 2 + (2x - 4) = -(x - 3) \]
\[ no \ solution \]
1-4 Spreadsheet Activity
Absolute Value Statements

You can use a spreadsheet to try several different values in an equation to help you determine whether the statement is sometimes, always, or never true. Remember that showing that a statement is true for some values does not prove that it is true for all values. However, finding one value for which a statement is false proves that it is not true for all values.

Determine whether \(|a + b| = |ca + cb|\) is sometimes, always, or never true.

Try a number of values for \(a, b,\) and \(c\) to determine whether the statement is true or false for each set of values.

Step 1 Use Columns A, B, and C for the values of \(a, b,\) and \(c\). Choose several sets of values including positive and negative numbers, and zero.

Step 2 Use Column D to test the equation. A formula such as \(C2*ABS(A2-1)\) in cell D2 returns TRUE if the equation is true.

Through observation of Column D, when \(c\) is negative the statement is not true. The absolute value statement, \(|a + b| = |ca + cb|\) is sometimes true; it is true only if \(c = 0\).

Exercises

Use a spreadsheet to determine whether each absolute value statement is sometimes, always, or never true.

1. For all real numbers \(a\) and \(b, a \neq 0, \ |ax + b| = 0\). Sometimes

2. If \(a\) and \(b\) are real numbers, then \(|a + b| = |a| + |b|\). Sometimes

3. If \(a\) and \(b\) are real numbers, then \(|a + b| = -x\). Never

4. If \(a\) and \(b\) are real numbers, then \(|a| - |b| = a - b\). Sometimes

5. If \(a\) and \(b\) are real numbers, then \(c|a + b| = c|a| + |b|\). Sometimes

1-5 Lesson Reading Guide
Solving Inequalities

Get Ready for the Lesson

Read the introduction to Lesson 1-5 in your textbook.

- Write an inequality comparing the number of minutes per month included in the two phone plans. \(150 < 400\) or \(400 > 150\)
- Suppose that in one month you use 475 minutes of airtime on your wireless phone. Find your monthly cost with each plan.

Plan 1: $65
Plan 2: $55

Which plan should you choose? Plan 2

Read the Lesson

1. There are several different ways to write or show inequalities. Write each of the following in interval notation.

   a. \([-3, -2] \cup [0, 1]\]
   b. \([-1, 2]\)
   c. \([-3, -2) \cup (1, 3]\)
   d. \([-4, -3) \cup [1, 2]\)

2. Show how you can write an inequality symbol followed by a number to describe each of the following situations.

   a. There are fewer than 600 students in the senior class. \(< 600\)
   b. A student may enroll in no more than six courses each semester. \(\leq 6\)
   c. To participate in a concert, you must be willing to attend at least ten rehearsals. \(\geq 10\)
   d. There is space for at most 165 students in the high school band. \(\leq 165\)

Remember What You Learned

3. One way to remember something is to explain it to another person. A common student error in solving inequalities is forgetting to reverse the inequality symbol when multiplying or dividing both sides of an inequality by a negative number. Suppose that your classmate is having trouble remembering this rule. How could you explain this rule to your classmate? Sample answer: Draw a number line. Plot two positive numbers, for example, 3 and 8. Then plot their additive inverses, \(-3\) and \(-8\). Write an inequality that compares the positive numbers and one that compares the negative numbers. Notice that \(8 > 3\), but \(-8 < -3\). The order changes when you multiply by \(-1\).
The city parking lot charges $2.50 for the first hour and $0.25 for each additional hour. If the most you want to pay for parking is $6.50, solve the inequality \(2.50 + 0.25x \leq 6.50\) to determine for how many hours you can park your car.

At most 17 hours

PLANNING For Exercises 2 and 3, use the following information. Ethan is reading a 482-page book for a book report due on Monday. He has already read 80 pages. He wants to figure out how many pages per hour he needs to read in order to finish the book in less than 6 hours.

2. Write an inequality to describe this situation. \(482 - 80 \leq 6n\) or \(6n \geq 402\).

3. Solve the inequality and interpret the solution. Ethan must read at least 70 pages per hour in order to finish the book in less than 6 hours.

BOWLING For Exercises 4 and 5, use the following information.

Four friends plan to spend Friday evening at the bowling alley. Three of the friends need to rent shoes for $3.50 per person. A string (game) of bowling costs $1.50 per person. If the friends pool their $40, how many strings can they afford to bowl?

4. Write an equation to describe this situation. \(3(3.50) + 4d \leq 40\).

5. Solve the inequality and interpret the solution. The friends can bowl at most 4 strings.
Answers (Lesson 1-5)

Solving Inequalities

Practice

Solve each inequality. Describe the solution set using set-builder or interval notation. Then, graph the solution set on a number line.

1. \( x + 3 < 10 \)
   \( x < 7 \)

2. \( 3n - 6 \leq 12 \)
   \( n \leq 6 \)

3. \( 3a - 10 > 20 \)
   \( a > 10 \)

4. \( 14 - 3m > 12 \)
   \( m < 2 \)

5. \( 5y + 3 \geq 12 \)
   \( y \geq 1.8 \)

6. \( n - 8 \leq 0 \)
   \( n \leq 8 \)

7. \( 7n - 3 \leq 19 \)
   \( n \leq 5 \)

8. \( 17x - 15 < 25 \)
   \( x < 2 \)

9. \( 10w - 10 > 10 \)
   \( w > 2 \)

10. \( 4v - 1 = 12 \)
    \( v = 3.5 \)

11. \( 2w + 1 = 5 \)
    \( w = 2 \)

12. \( 3x - 2 = 7 \)
    \( x = 3 \)

13. \( 2.5 = 1.5 + n \)
    \( n = 1 \)

14. \( 4n - 3 = 2n - 3 \)
    \( n = 0 \)

15. \( 3n - 2 > n + 2 \)
    \( n > 2 \)

16. \( 4n - 2 < n + 3 \)
    \( n < 1 \)

17. \( 2n - 3 > n - 1 \)
    \( n > 2 \)

18. \( 4n - 2 < n + 3 \)
    \( n < 1 \)

19. \( 2n - 3 > n - 1 \)
    \( n > 2 \)

20. \( 3n - 2 < n + 2 \)
    \( n < 1 \)

Define a variable and write an inequality for each problem. Then solve.

1. The difference of three times a number and 16 is at least 8.
   \( 3n - 16 \geq 8 \)
   \( n \geq 8 \)

2. The sum of seven and a number is less than ten.
   \( 7 + n < 10 \)
   \( n < 3 \)

3. The product of a number and 16 is more than twice the same number.
   \( 16n > 2n \)
   \( n > 0 \)

4. The sum of a number and -3 is less than 5.5 times that same number.
   \( n + (-3) < 5.5n \)
   \( n < 6 \)

5. The difference of two numbers is at least 12.
   \( n - m \geq 12 \)
   \( n \geq m + 12 \)

6. The sum of a number and -2 is at least 10.
   \( n + (-2) \geq 10 \)
   \( n \geq 12 \)

7. The difference of two numbers is at least 12.
   \( n - m \geq 12 \)
   \( n \geq m + 12 \)

8. The sum of a number and -2 is at least 10.
   \( n + (-2) \geq 10 \)
   \( n \geq 12 \)

9. The product of a number and -2 is at least 10.
   \( n \cdot (-2) \geq 10 \)
   \( n \leq -5 \)

10. The sum of a number and -2 is at least 10.
    \( n + (-2) \geq 10 \)
    \( n \geq 12 \)

11. The difference of two numbers is at least 12.
    \( n - m \geq 12 \)
    \( n \geq m + 12 \)

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    \( n \geq 12 \)

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    \( n \geq 12 \)

15. The difference of two numbers is at least 12.
    \( n - m \geq 12 \)
    \( n \geq m + 12 \)

16. The sum of a number and -2 is at least 10.
    \( n + (-2) \geq 10 \)
    \( n \geq 12 \)

17. The difference of two numbers is at least 12.
    \( n - m \geq 12 \)
    \( n \geq m + 12 \)

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20. The sum of a number and -2 is at least 10.
    \( n + (-2) \geq 10 \)
    \( n \geq 12 \)
1. PANDAS An adult panda bear will eat at least 20 pounds of bamboo every day. Write an inequality that expresses this situation.

\[ b \geq 20 \]

2. PARTY FAVORS Janelle would like to give a party bag to every person who is coming to her party. The cost of the party bag is $7 per person. Write an inequality that describes the number of people \( P \) that she can invite if Janelle has \( D \) dollars to spend on the party bags.

\[ P \cdot 7 \leq D \]

3. INCOME Manuel takes a job translating English instruction manuals to Spanish. He will receive $15 per page plus $100 per month. Manuel would like to work for 3 months during the summer and make at least $1,500. Write and solve an inequality to find the minimum number of pages Manuel must translate in order to reach his goal.

\[ 3 \times 15x + 300 \geq 1500 \]

\[ P \geq 80; \] Manuel must translate at least 80 pages.

4. FINDING THE ERROR The sample below shows how Brandon solved 5 \( < -2x - 7 \). Study his solution and determine if it is correct. Explain your reasoning.

\[ 5 < -2x - 7 \]
\[ \Rightarrow -2x < -12 \]
\[ \Rightarrow x > 6 \]

It is incorrect. From step 2 to step 3, Brandon must change the direction of the inequality because he is dividing by a negative number.

The correct answer is \( x < -6 \).

5. CARNIVALS For Exercises 5–7, use the following information.

On a Ferris wheel at a carnival, only two people per car are allowed. The two people together cannot weigh more than 300 pounds. Let \( x \) and \( y \) be the weights of the people.

5. Write an inequality that describes the weight limitation in terms of \( x \) and \( y \).

\[ x + y \leq 300 \]

6. Write an inequality that describes the limit on the average weight \( a \) of the two riders.

\[ a \leq 150 \]

7. Ron and his father want to go on the ride together. Ron's father weighs 175 pounds. What is the maximum weight Ron can be for the two to be allowed on the ride? 125 pounds

1-5 Word Problem Practice
Solving Inequalities

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1-5 Enrichment
Equivalence Relations

A relation \( R \) on a set \( A \) is an equivalence relation if it has the following properties.

 Reflexive Property For any element \( a \) of set \( A \), \( a \ R \ a \).
 Symmetric Property For all elements \( a \) and \( b \) of set \( A \), if \( a \ R \ b \), then \( b \ R \ a \).
 Transitive Property For all elements \( a \), \( b \), and \( c \) of set \( A \), if \( a \ R \ b \) and \( b \ R \ c \), then \( a \ R \ c \).

Equality on the set of all real numbers is reflexive, symmetric, and transitive. Therefore, it is an equivalence relation.

In each of the following, a relation and a set are given. Write yes if the relation is an equivalence relation on the given set. If it is not, tell which of the properties it fails to exhibit.

1. \( < \), \{all numbers\} no; reflexive, symmetric
2. \( = \), \{all triangles in a plane\} yes
3. is the sister of, \{all women in Tennessee\} no; reflexive
4. \( \geq \), \{all numbers\} no; symmetric
5. is a factor of, \{all nonzero integers\} no; symmetric
6. \( \\bot \), \{all polygons in a plane\} yes
7. is the spouse of, \{all people in Roanoke, Virginia\} no; reflexive, transitive
8. \( \perp \), \{all lines in a plane\} no; reflexive, transitive
9. is the square of, \{all numbers\} no; reflexive, symmetric, transitive
10. is the multiple of, \{all integers\} no; symmetric
11. \( < \), \{all numbers\} no; reflexive, symmetric, transitive
12. has the same color eyes as, \{all members of the Cleveland Symphony Orchestra\} yes
13. is the greatest integer not greater than, \{all numbers\} no; reflexive, symmetric, transitive
14. is the greatest integer not greater than, \{all integers\} yes
**Answers (Lesson 1-6)**

**Solving Compound and Absolute Value Inequalities**

1. **Compound Inequalities**
   - A compound inequality consists of two inequalities joined by the word and or or.
   - To solve a compound inequality, you must solve each part separately.

2. **Absolute Value Inequalities**
   - The graph is the union of solution sets of two inequalities.
   - The number line graph will show two disconnected intervals with arrows going in opposite directions.

3. **Writing Compound Inequalities**
   - A compound inequality consists of two inequalities joined by the word and or or.
   - To solve a compound inequality, you must solve each part separately.

4. **Example:**
   - Graph the solution set of the compound inequality.
   - The graph is the intersection of solution sets of two inequalities.
   - The number line graph will show a single interval.

5. **Example:**
   - Write a compound inequality that says, "or an inequality.
   - The number line graph will show two disconnected intervals with arrows going in opposite directions.

6. **Example:**
   - Solve the compound inequality.
   - The number line graph will show two disconnected intervals with arrows going in opposite directions.

7. **Example:**
   - Solve the compound inequality.
   - The number line graph will show two disconnected intervals with arrows going in opposite directions.

8. **Example:**
   - Solve the compound inequality.
   - The number line graph will show two disconnected intervals with arrows going in opposite directions.
Answers (Lesson 1-6)

Skills Practice

Solving Compound and Absolute Value Inequalities

1. Write an absolute value inequality for each of the following. Then graph the solution set on a number line.

   a. all numbers less than 5 and greater than -3
      \[ |x - 5| < 3 \]
      \[ -3 < x < 8 \]

   b. all numbers greater than 10 and less than 20
      \[ |x| > 10 \]
      \[ x < -10 \text{ or } x > 20 \]

2. Write an absolute value inequality for each graph.

   a. \[ |x| < 4 \]
   b. \[ |x| > 3 \]
   c. \[ |x| < 5 \]
   d. \[ |x| > 2 \]
   e. \[ |x| < 6 \]
   f. \[ |x| > 5 \]

3. Solve each inequality. Graph the solution set on a number line.

   a. \[ |x - 4| < 3 \]
      \[ 1 < x < 7 \]

   b. \[ |2x + 1| > 3 \]
      \[ x < -2 \text{ or } x > 1 \]

   c. \[ |x - 2| > 4 \]
      \[ x < -2 \text{ or } x > 6 \]

   d. \[ |x - 3| < 2 \]
      \[ 1 < x < 5 \]

   e. \[ |x - 10| > 8 \]
      \[ x < 2 \text{ or } x > 18 \]

   f. \[ |x - 5| < 6 \]
      \[ -1 < x < 11 \]

   g. \[ |x + 2| > 4 \]
      \[ x < -6 \text{ or } x > 2 \]

   h. \[ |x - 6| < 3 \]
      \[ 3 < x < 9 \]

4. Graph the solution set on a number line.

   a. \[ |x - 3| < 8 \]
   b. \[ |x + 3| > 8 \]
   c. \[ |x - 2| = 5 \]
   d. \[ |x + 1| < 7 \]
   e. \[ |x - 2| > 1 \]
   f. \[ |x + 3| < 1 \]

Exercises

1. Solve each inequality. Graph the solution set on a number line.

   a. \[ |x + 4| < 8 \]
      \[ -12 < x < 4 \]

   b. \[ |x - 1| > 2 \]
      \[ x < -1 \text{ or } x > 3 \]

   c. \[ |x + 3| < 5 \]
      \[ -8 < x < 2 \]

   d. \[ |x - 2| > 6 \]
      \[ x < -4 \text{ or } x > 8 \]

   e. \[ |x + 1| < 3 \]
      \[ -4 < x < 2 \]

   f. \[ |x - 4| > 2 \]
      \[ x < 2 \text{ or } x > 6 \]

2. Solve each inequality. Graph the solution set on a number line.

   a. \[ |x - 1| < 5 \]
      \[ 0 < x < 10 \]

   b. \[ |x + 2| > 4 \]
      \[ x < -6 \text{ or } x > 2 \]

   c. \[ |x - 3| < 6 \]
      \[ -3 < x < 9 \]

   d. \[ |x + 4| < 8 \]
      \[ -12 < x < 4 \]

   e. \[ |x - 1| > 3 \]
      \[ x < -2 \text{ or } x > 4 \]

   f. \[ |x + 2| < 6 \]
      \[ -8 < x < 4 \]

   g. \[ |x - 3| > 4 \]
      \[ x < -1 \text{ or } x > 7 \]

   h. \[ |x + 1| < 5 \]
      \[ -6 < x < 4 \]

   i. \[ |x - 2| > 6 \]
      \[ x < -4 \text{ or } x > 8 \]

   j. \[ |x + 2| < 4 \]
      \[ -6 < x < 2 \]

   k. \[ |x - 3| < 5 \]
      \[ -2 < x < 8 \]

   l. \[ |x + 1| > 7 \]
      \[ x < -8 \text{ or } x > 6 \]

   m. \[ |x - 2| > 8 \]
      \[ x < -6 \text{ or } x > 10 \]

   n. \[ |x + 2| < 10 \]
      \[ -12 < x < 8 \]

   o. \[ |x - 3| < 10 \]
      \[ -7 < x < 13 \]

   p. \[ |x + 1| > 5 \]
      \[ x < -6 \text{ or } x > 4 \]

   q. \[ |x - 2| = 4 \]
      \[ x = 6 \text{ or } x = -2 \]

   r. \[ |x + 3| = 5 \]
      \[ x = 2 \text{ or } x = -8 \]

   s. \[ |x - 4| = 2 \]
      \[ x = 2 \text{ or } x = 6 \]

   t. \[ |x + 1| = 3 \]
      \[ x = 2 \text{ or } x = -4 \]

   u. \[ |x - 3| = 7 \]
      \[ x = 10 \text{ or } x = -4 \]

   v. \[ |x + 2| = 1 \]
      \[ x = -3 \text{ or } x = -1 \]

   w. \[ |x - 4| = 0 \]
      \[ x = 4 \]
Write an absolute value inequality for each of the following. Then graph the solution set on a number line.

1. all numbers greater than 4 or less than -4 \(| n > 4\) \(| n < -4\)

2. all numbers between -1.5 and 1.5, including -1.5 and 1.5 \(| -1.5 \leq n \leq 1.5\)

Write an absolute value inequality for each graph.

3. \(| n \geq 10\) \(| n \leq -10\) \(| 0 \leq n \leq 20\)

4. \(| n < \frac{4}{3}\) \(| n > \frac{4}{3}\)

Solve each inequality. Graph the solution set on a number line.

5. \(-8 \leq 3y - 20 < 52\) \((y | 4 \leq y < 24\)

6. \(3.5x - 2) < 24\) or \(6x - 4 > 4 + 5x\) \((x | x < 2\) or \(x > 8\)

7. \(2x - 3 > 15\) or \(3x - 7 < 17\) \((x | x > -2)\) \((x | x < 3)\)

8. \(15 - 5x \leq 0\) and \(5x + 6 \geq -14\) \((x | x \geq 3)\)

9. \(| 2w | = 5\) \((w | w \leq \frac{5}{2}\) or \(w \geq \frac{5}{2}\)

10. \(| y + 5 | < 2\) \((x | -7 < x < -3)\)

11. \(| x - 8 | \geq 3\) \((x | x \geq 11)\)

12. \(| 2x - 2 | \leq 3\) \((x | -\frac{1}{2} \leq x \leq \frac{5}{2})\)

13. \(| 2x + 2 | - 7 \leq -5\) \((x | -2 \leq x \leq 0)\)

14. \(| z | > x - 1\) \((x \) all real numbers\)

15. \(| 3y + 5 | \leq -2\) \((\emptyset)\)

16. \(| 3y - 2 | - 2 < 1\) \((n | -\frac{1}{3} < n \leq \frac{5}{3})\)

17. RAINFALL In 90% of the last 30 years, the rainfall at Shell Beach has varied no more than 6.5 inches from its mean value of 24 inches. Write and solve an absolute value inequality to describe the rainfall in the other 10% of the last 30 years.

\(| r - 24 | > 6.5\) \(| r < 17.5\) or \(r > 30.5)\)

18. MANUFACTURING A company's guidelines call for each can of soup produced not to vary from its stated volume of 14.5 fluid ounces by more than 0.08 ounces. Write and solve an absolute value inequality to describe acceptable can volumes.

\(| v - 14.5 | \leq 0.08\) \(| v | 14.42 \leq v \leq 14.58\)

AQUARIUM The depth of a tank for dolphins satisfies \(| d - 50 | < 5\). Write this as a compound inequality that does not involve the absolute value function. \(45 < d < 55\)

HIKING For a hiking trip, everybody must bring at least one backpack. However, because of space limitations, nobody is allowed to bring more than two backpacks. Let \(n\) be the number of people going on the hiking trip and \(b\) be the number of backpacks allowed. Write a compound inequality that describes how \(b\) and \(n\) are related. \(n \leq b \leq 2n\)

CONCERT Jacinta is organizing a large fund-raiser concert in a space with a maximum capacity of 30,000 people. Her goal is to raise at least $100,000. Tickets cost $20 per person. Jacinta spends $50,000 to put the event together. Write and solve a compound inequality that describes \(N\), the number of attendees needed to achieve Jacinta's goal.

\(20N = 50,000 \leq 100,000\) and \(N \leq 10,000\); The attendance must be between 7,500 and 10,000 people, inclusive.

NUMBERS Amy is thinking of two numbers \(a\) and \(b\). The sum of the two numbers must be within 10 units of zero. If \(a\) is between -100 and 100, write a compound inequality that describes the possible values of \(b\). \(-110 < b < 110\)

AIRLINE BAGGAGE For Exercises 5-7, use the following information.

An airline company has a size limitation for carry-on luggage. The limitation states that the sum of the length, width, and height of the suitcase must not exceed 45 inches.

1. Write an inequality that describes the airline's carry-on size limitation. \(h + w + l \leq 45\)

2. Write a compound inequality to describe acceptable can volumes. \(| v - 14.5 | \leq 0.08\) \(| v | 14.42 \leq v \leq 14.58\)

3. Write a compound inequality for a using parts A and B. Find the maximum and minimum values for \(s\). \(s \geq 12\) and \(s \leq 25\) and \(s = 12\) at most 15
Conjunctions and Disjunctions

An absolute value inequality may be solved as a compound sentence.

Example 1: Solve \(|2x| < 10\).

\(|2x| < 10\) means \(2x < 10\) and \(2x > -10\).

Solve each inequality: \(x < 5\) and \(x > -5\).

Every solution for \(|2x| < 10\) is a replacement for \(x\) that makes both \(x < 5\) and \(x > -5\) true.

A compound sentence that combines two statements by the word *and* is a conjunction.

Example 2: Solve \(|3x - 7| = 11\).

\(|3x - 7| = 11\) means \(3x - 7 = 11\) or \(3x - 7 = -11\).

Solve each inequality: \(3x \geq 18\) or \(3x \leq 4\)

\(x \geq 6\) or \(x \leq \frac{4}{3}\)

Every solution for the inequality is a replacement for \(x\) that makes either \(x \geq 6\) or \(x \leq \frac{4}{3}\) true.

A compound sentence that combines two statements by the word *or* is a disjunction.

Solve each inequality. Then write whether the solution is a conjunction or disjunction.

1. \(|4x| > 24\) \(x > 6\) or \(x < -6\); disjunction
2. \(|x - 7| \leq 8\) \(x \leq 15\) and \(x \geq -1\); conjunction
3. \(|2x + 5| < 1\) \(x - 1 = 1\)
4. \(|x + 1| \leq 7\) \(x = -2\) and \(x > -3\); conjunction
5. \(|3 - x| \leq 5\) \(|x - 7 - |2x| > 5\)
6. \(|x - 1| \leq 4\) \(|x - 1| \leq 0\); disjunction
7. \(|x - 1| \leq 7\) \(|x - 1| > 3\); conjunction
8. \(|x - 1| < 4\) \(|x - 1| < 4\)
9. \(|8 - x| > 2\) \(|x - 12\) or \(x \leq -16\); disjunction
10. \(|x - 1| > 3\) \(x < 16\) and \(x > -8\); conjunction

\(x < 6\) or \(x > 10\); disjunction

\(x \geq 1\) and \(x \leq 4\); conjunction