Exponential Growth & Decay - Formula 2

\[ y = ae^{kt} \]

*Used in scientific applications - half-life/radioactive decay, bacteria growth, etc.

\[ a = \text{Initial Amount} \]
\[ k = \text{constant of proportionality} \]
\[ t = \text{time} \]
Example 1: The half-life of Sodium-22 is 2.6 years. Find the value of $k$ and the equation.

\[ k = -\frac{\ln(2)}{2.6} \]

\[ -0.6931 = \frac{k(2.6)}{2.6} \]

\[ y = ae^{-0.2666t} \]

A geologist examining a meteorite estimates that it contains only about 10% as much Sodium-22 as it would have contained when it reached Earth's surface. How long ago did the meteorite reach the surface of Earth?

\[ 0.10 = e^{-0.2666t} \]

\[ \ln(0.10) = \ln(e^{-0.2666t}) \]

\[ -2.3026 = -0.2666t \]

\[ \frac{-2.3026}{-0.2666} = t \]

\[ 8.6 \text{ years} \]

Write an exponential growth equation of the form \( y = ae^{kt} \) for Fayette County, where \( t \) is the number of years after 2000.

\[
\frac{268,080}{260,512} = \frac{260,512e^{5k}}{260,512} \quad \frac{0.0291}{e^{5k}} = 0.0287 = \frac{5k}{5} \quad 0.0057 = k
\]

Use your equation to predict the population of Fayette County in 2015.

\[
y = 260,512e^{0.0057t} \\
y = 260,512e^{0.0057(15)} \\
y = 260,512e^{0.0855} \\
y = 260,512(1.0893) \\
y = 283,176
\]
Example 3: The population, P, in thousands, of a city can be modeled by the equation, where $t$ is the time in years. In how many years will the population of the city be 120,000?

\[ P = 80e^{0.015t} \]

\[
\frac{120,000}{80} = \frac{80e^{0.015t}}{80}
\]

\[ \ln(1500) = \ln(e^{0.015t}) \]

\[
7.3132 = \frac{0.015t}{0.015}
\]

\[ 487.5 = t \text{ years} \]