Measuring Distance and Displacement

Background Information

Vectors have many uses. For example, you can use vectors to describe the distance an object travels and the displacement that results from an object’s movement. Before you can make use of vectors, you must first select a frame of reference.

To use vectors to describe an object’s position or movement on a flat surface, you must first define a frame of reference that includes the origin—a specific point that does not move. Two imaginary lines, or axes, that pass through the origin at right angles are then chosen, as shown in Figure 1. These two lines are the \( x \)-axis and the \( y \)-axis.

Using the axes, you can describe the position of an object in terms of its \( x \)- and \( y \)-coordinates, for example, the point \((4, 8)\) on a graph. Note that the coordinates of the origin are \((0, 0)\).

Figure 1 shows that you can also define a vector to describe an object’s position by drawing an arrow from the origin to the object’s position. The object’s coordinates \((7, 2)\) determine the length and direction of the vector. Figure 2 shows that any vector in the frame of reference can be broken down into \( x \)- and \( y \)-components. Therefore, any vector is also the resultant vector of its own \( x \)- and \( y \)-components. Resultant vectors are also used to represent the result of vector addition.
You can use a similar method to define a vector describing an object’s displacement. To do this, draw a vector from the object’s starting point to its ending point. Figure 3 shows the x- and y-coordinates of an object’s starting and ending points. Note that the x- and y-components of the displacement vector are simply the difference between the x- and y-coordinates of the starting and ending points.

In this investigation, you will compare two methods of determining the length of a displacement vector.

**Problem**
How can vectors be used to determine displacement?

**Pre-Lab Discussion**
*Read the entire investigation. Then, work with a partner to answer the following questions.*

1. How will you determine the x- and y-coordinates of each position?

2. How will you determine the x- and y-components of the displacement vector?

3. How will you calculate the length of the displacement vector?

4. How will you measure the length of the displacement vector?
**Materials** *(per group)*

- masking tape
- meter stick
- calculator
- string

**Safety**

Put on safety goggles. Use caution to avoid bumping into people or objects when moving around the room. Note all safety alert symbols next to the steps in the Procedure and review the meaning of each symbol by referring to the Safety Symbols on page xiii.

**Procedure**

1. Work with a classmate. Mark a dot on a small piece of masking tape. Mark the origin of your frame of reference by sticking a piece of masking tape on the floor, away from furniture and other obstacles.

2. Use the width of the classroom as the $x$-direction and the length of the classroom as the $y$-direction. Attach a 2-meter strip of tape to the floor, running from the origin in the $x$ direction. This is the $x$-axis. Attach a second 2-meter strip of tape to the floor, running from the origin in the $y$ direction. This is the $y$-axis. Note that the $x$- and $y$-axes should be at right angles to each other.

3. Select a point 1 to 4 meters from the origin as your starting point. Mark this point by sticking a piece of masking tape on the floor and marking a dot on it. Label this piece of tape *Start*.

4. Walk from the starting point to another point 1 to 4 meters from the origin. Mark this point by sticking a piece of masking tape on the floor and marking a dot on it. Label this piece of tape *End*.

5. Use a meter stick to measure the $x$-coordinate of the starting point to the nearest centimeter, as shown in Figure 4. Be careful to measure parallel to the $x$-axis. Record your measurement in the data table.

6. Repeat Step 5, measuring parallel to the $y$-axis, to determine the $y$-coordinate of the starting point. Record your measurement in the data table.
7. Measure and record the $x$- and $y$-coordinates of the ending point in the same way that you determined the coordinates of the starting point.

8. Determine the $x$-component of the displacement vector by subtracting the $x$-coordinate of the starting point from the $x$-coordinate of the ending point. Record this value in the data table as the vector component in the $x$-direction.

9. Repeat Step 8 with the $y$-coordinates of the starting and ending points to calculate the $y$-component of the displacement vector. Record this value in the data table as the vector component in the $y$-direction.

10. Calculate and record the square of the vector component in the $x$-direction. Calculate and record the square of the vector component in the $y$-direction.

11. Use the following formula to calculate the length of the displacement vector

\[ L = \sqrt{x^2 + y^2} \]

where $L$ is the length of the displacement vector, and $x$ and $y$ are the $x$- and $y$-components of the displacement vector. Record this value in the data table as the vector length of the displacement vector.

12. Working with your partner, stretch a string from the starting point to the ending point. While holding the string in this position, mark the string at both points. Use a meter stick to measure the distance between the two marks on the string. Record this measurement in the data table as the measured vector length of the displacement vector.

**Observations**

<table>
<thead>
<tr>
<th>Direction</th>
<th>Coordinates of Starting Point (cm)</th>
<th>Coordinates of Ending Point (cm)</th>
<th>Displacement Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>Calculated</td>
<td>Measured</td>
<td></td>
</tr>
<tr>
<td>$y$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Analysis and Conclusions

1. Measuring  What tool can you use to measure the distance moved by an object?

2. Measuring  How can you measure the distance an object has moved?

3. Measuring  How can you measure the magnitude of the displacement of an object?

4. Calculating  How can you calculate the displacement of an object?

5. Controlling Variables  How could using a large book as the displaced object produce significant error in your results?

6. Comparing and Contrasting  How did the calculated length of the displacement vector compare with the measured length of the displacement vector?
7. **Inferring**  Describe a condition in which it would be impossible to actually measure a displacement.

8. **Comparing and Contrasting**  How does the distance moved by an object between its starting point and ending point compare to the displacement vector between the same points?

**Go Further**

In this investigation, you determined the distance between two points by measuring and calculating the displacement vector. How could you determine displacement by using a graph? Write a procedure you would follow to answer this question. Have your teacher approve your procedure before you carry out the investigation. How does the displacement determined by this graphing method compare with the actual measurement of the displacement?